The Human Capital - Reproductive Capital Tradeoff in Marriage Market Matching

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For much of recent history, the relationship between women's human capital and men's income was non-monotonic: while college-educated women married richer spouses than high school-educated women, graduate-educated women married poorer spouses than college-educated women. This can be rationalized by a bi-dimensional matching framework where women's human capital is negatively correlated with another valuable trait, fertility, or "reproductive capital." I use a transferable utility matching model to show that bi-dimensionality with negatively correlated traits can produce non-monotonicity in income matching with a sufficiently high-income distribution of men. A simulation of the model using Census fertility and income data shows that it can predict the recent transition to more assortative matching as the skill premium has increased, assisted reproduction technologies have emerged, and desired family sizes have fallen.

JEL Codes: J12, J13, J16, D13, C78

1 Introduction

It has long been suspected that higher earning may not always yield women wealthier mates. Early theory on marriage markets predicted that in fact matching should be negative assortative on income, due to returns to specialization [Becker, 1973]. Recent research suggests that earning high income itself could make women less desirable to potential partners [Bursztyn et al., 2017, Bertrand et al., 2015]. And yet matching is generally *positive* assortative on income, and most literature shows it has become more so over time [Chiappori et al., 2017b, Hurder, 2013, Greenwood et al., 2016, 2014, Fernandez et al., 2005, Schwartz and Mare, 2005].[1]

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[1]Note that Gihleb and Lang [2016] and Eika et al. [2019] do not find increasing assortativeness over time.

In this paper, I posit that underlying these apparent contradictions is the fact that human capital investments that yield greater income may also decrease another desirable marriage market trait, "reproductive capital." Women's fertility decreases with age, and human capital investments that increase income delay marriage and childbearing and increase spacing between births. I first show that older age at marriage is linked to lower spousal income for women, aligning with experimental findings that men value women's age, through the channel of fertility [Low, 2021]. I then document, for the first time, that husband's income has historically exhibited a non-monotonic pattern in wife's education: additional education up to a college degree was associated with increased spousal income, but education beyond college was associated with decreased spousal income. This pattern cannot be rationalized by a traditional unidimensional model, but can be easily explained by a bi-dimensional model where income is negatively correlated with fertility.

I outline a transferable utility matching model between men characterized by income and women characterized by income and fertility. A latent "human capital type" impacts both income and fertility. This contributes to a growing literature showing that truly multidimensional models, as opposed to index frameworks, may be crucial in understanding matching patterns, since valuations of non-income traits likely vary with income [Chiappori et al., 2017a, Galichon and Salanié, 2015, Coles and Francesconi, 2011, Lindenlaub and Postel-Vinay, 2017, Dupuy and Galichon, 2014, Galichon et al., 2019, Coles and Francesconi, 2019]. I demonstrate that with a surplus function that is supermodular in both incomes and income and fertility, non-monotonic matching on incomes can appear. The stable match will depend on the tradeoff between human and reproductive capital in women's type distribution, relative to men's income distribution. I provide a simple condition such that there always exists a man rich enough that he prefers a higher fertility but poorer woman to a richer and less fertile woman.

Women's educational investment will then depend on the matching penalty, aligning with theoretical and empirical work showing fertility concerns can affect career investments [Siow, 1998, Dessy and Djebbari, 2010, Zhang, 2021, Gershoni and Low, 2021b], but highlighting the equilibrium matching channel in addition to personal utility loss from lower fertility. Nonetheless, it is possible to sustain an equilibrium where women invest in human capital despite worse matching outcomes, because they value the wage returns over the marriage market penalty.[2]

[2] This contributes an example where investments can affect multiple dimensions to the literature on premarital investments [Iyigun and Walsh, 2007, Peters and Siow, 2002, Lafortune, 2013, Dizdar, 2018, Mailath et al., 2013, Cole et al., 2001, Nöldeke and Samuelson, 2015, Mailath et al., 2017].

Finally, I simulate the model using Census data on income and fertility over time, and demonstrate that it can match the evolution of historical patterns. The convergence between highly educated and college educated women's fertility rates as average family sizes fell can produce the shift from non-monotonic to assortative mating, matching a broader "reversal of fortune" for educated women on the marriage market [Fry, 2010, Rose, 2005, Isen and Stevenson, 2010, Bertrand et al., 2020]. Moreover, the model can match the increase in women pursuing graduate education over time.

Together, these results demonstrate that while we may presume women value fertility personally, it also affects them economically. This paper uses different levels of education to demarcate human capital investments, standing in for the more general problem of the tradeoff between career investments and fertility, which is of first-order concern to women.[3] Individuals, policymakers, and firms may be able to use a better understanding of this tradeoff to blunt the impact of reproductive capital's decline.

The remainder of the paper proceeds as follows: Section 2 documents stylized facts, Section 3 develops a model that incorporates fertility in the marital surplus function, Section 4 simulates the model, and Section 5 concludes.

2 Stylized Facts

This section establishes two new stylized facts. First, that older age at first marriage for women is associated with poorer spouses. Second, that women's human capital, which increases earnings but decreases fertility, has historically been non-monotonically related to spousal income.

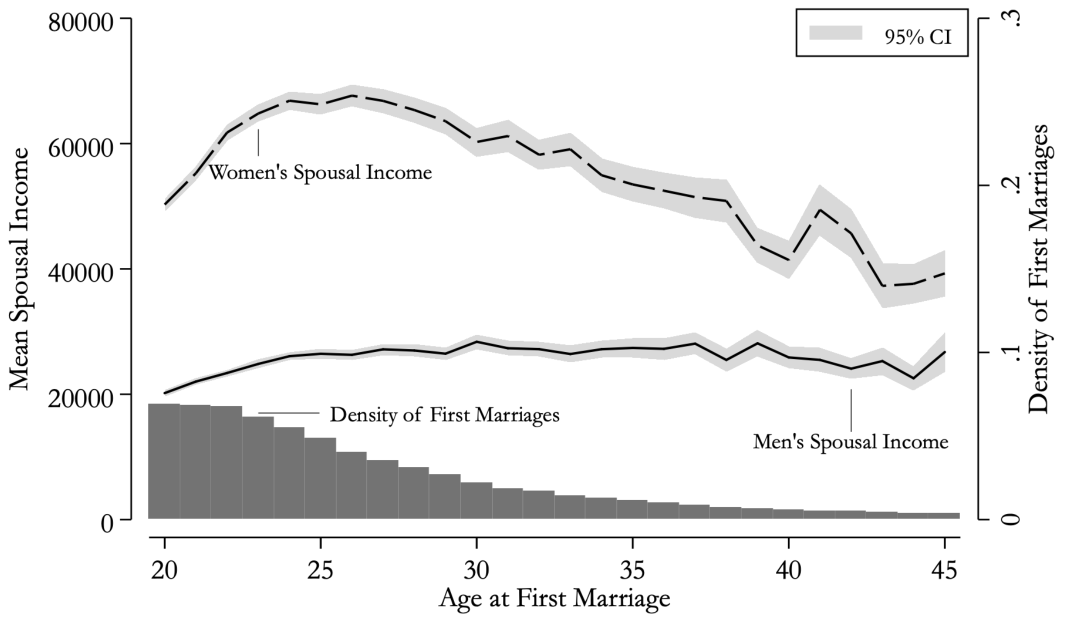
First, Figure 1 shows that for women over the age of around 25, each year older that they marry is associated with lower spousal income.[4] Although the negative relationship between women's age and spousal income is only correlational, the fact that individuals who marry later tend to be positively selected makes it suggestive of a negative impact of age on marriage market outcomes.[5]

[3] For evidence that it is difficult to co-process career investments and fertility, see Goldin and Katz [2002], Bailey [2006], Bailey et al. [2012], Adda et al. [2017], Kleven et al. [2019], and Gershoni and Low [2021a] for evidence that future reproductive time horizons drive young women's decision-making.

[4] This pattern is shown in women 46-55 at the time of the 2010 American Community Survey, so that marriages up to age 45 can be shown. To verify that neither the selection of ages nor very late marriages are driving the pattern, Appendix Figure A1 shows the same pattern for women currently age 36-45, married up to age 35.

[5] One might worry the pattern stems from unobservable selection, if women who marry later are "leftover". However, the pattern of marriage volume makes this unlikely, since the bulk of marriages—and thus the largest possible sorting—occurs before the decline in husband's income begins, as shown by the density graph. Zhang [2021] additionally notes that the selection of men who marry late tends to be negative, but we do not see the

Figure 1: Spousal Income by Age at Marriage



Notes: Lines represent the average spousal income by age at marriage for women versus men currently in their first marriage. Income is current spousal income, for individuals currently 46-55 years old. Bars represent the portion of all women's marriages occurring at that age, to check whether selection is driving the effect. Source: 2010 American Community Survey (1 percent sample).

This aligns with evidence from an incentive-compatible experiment in Low [2021] that men have a negative preference for women's age when it is randomly assigned to dating profiles. This preference is driven by men with accurate knowledge of the fertility-age tradeoff and who have no children themselves, suggests it is driven by fertility concerns.

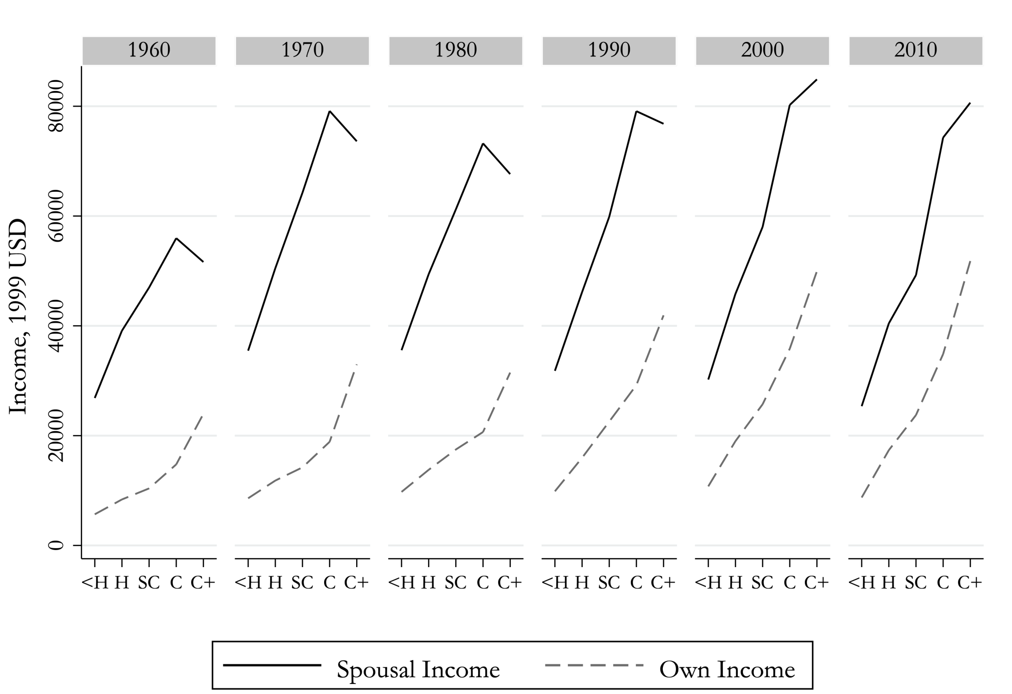
If men indeed value fertility as a marriage market trait, it suggests that time-consuming human capital investments would be a double-edged sword for women: on the one hand, human capital carries higher earning, a presumably positive attribute likely to help attract a high-income spouse. On the other hand, income-increasing investments take time, decreasing what could be another valuable asset on the marriage market, reproductive capital.

While much empirical work categorizes all women with college degrees as "college plus," the "reproductive capital" hypothesis suggests women with college degrees and graduate degrees may have very different marriage market outcomes, since women with college degrees only could still marry quite young and have large families. Moreover, graduate degrees are correlated with the types of high-investment careers that may continue to interfere with time to have children: the tenure track, the partner track, surgical residencies, and climbing the corporate ladder.

same declining spousal income for men.

Figure 2 shows that when graduate and college education are graphed separately, there has historically been a non-monotonic relationship between women's education and men's income. All levels of education prior to a graduate degree are associated with higher income, whereas graduate degrees are associated with lower spousal income, up until the 1990s.[6] The relationship between education and own income, by contrast, is monotonic, and in fact own income increases the most steeply between college and graduate education, in all years.

Figure 2: Non-monotonicity in spousal income by wife's education level



Notes: Income of spouse based on wife's education level. <H=less than high school, H=high school grad, SC=some college, C=college grad, C+=graduate degree. Source: 1 percent Census data from 1960, 1970, 1980, and 1990 and ACS data from 2000 and 2010. Sample consists of women, ages 41-50 years old (married, for spousal income).

The spousal matching penalty to graduate education is statistically significant, as shown in Appendix Table A1, and economically meaningful. In 1970 and 1980, a highly educated woman was married to a man making about $6,000 less than a college educated woman, despite making over $10,000 more herself. This penalty appears up to the 2000 data, when a monotonic relationship emerges, although there is still less steep growth between the spousal income of a highly and college educated woman than between other educational levels.

[6] US Census and ACS 1% sample, restricted to women 41-50, so that the vast majority of first marriage activity and educational investments have already taken place by the time they are observed. This graph includes all marriages, rather than only first, as number of times married is unavailable in 1990 and 2000. To ensure second marriages are not driving the pattern, Appendix Figure A2 repeats the analysis for first marriages only, omitting 1990 and 2000, and finds entirely comparable patterns.

Unidimensional models fail to match this non-monotonicity. Division of labor, and thus substitutability between men's and women's incomes, could explain the negative relationship between education and spouse's income for college and graduate educated women, but not the positive relationship at other education levels. Complementarity in spouses' incomes, social class, or education could explain the positive relationship in most of the data, but not the apparent "penalty" to graduate education.

Moreover, while a relative increase in marriage rates (and decrease in divorce rates) for educated women has been noted in the literature, I show in Appendix Figure A3 that these changes were actually driven specifically by graduate-educated women. College educated women have historically had comparable marriage and divorce rates to women with less education. Only highly educated women previously married substantially less and divorced more, and have therefore driven the recent reversal.

These facts suggest a second factor that is decreasing in education, even as income rises. Census data shows that highly educated women have substantially fewer children in 1970 than college educated women, as shown in Table 1. The difference between those with college degrees and those with high school education or some college is substantially smaller, especially when it comes to the proportion with greater than 4 children. Note that while highly educated women marry older than other educational levels, the difference in fertility may also be driven by further delays in childbearing, potentially linked to post-education career investments.[7]

Thus, in the next section I develop a model of matching where human capital investments increase income, but decrease fertility, and explore its implications for the duality of educational investments for women.

3 Theoretical Framework

If men value women's fertility, it will have consequences for matching patterns, as well as women's willingness to invest in human capital. This section studies this using a bi-dimensional, transferable

[7] While other elements in addition to fertility could be negatively correlated with income, they may be less likely to be complementary with income, which is a key driver of the model's ability to produce non-monotonic income matching patterns. I explore some of these alternative explanations in Appendix C.4. While I cannot rule out that highly educated women have lower tastes for children, rather than lower ability to have children, this nonetheless implies a male valuation of fertility. However, suggestive evidence in Appendix C.4 shows that selection is unlikely to entirely be the driver of the matching patterns we observe, since the spousal income penalty moves little during a time when the number of women seeking graduate degrees doubled. Even if the second factor is something other than fertility, the key point is that the data reflect a duality of human capital investments for women that does not exist for men.

Table 1: Income, Spousal Income, Age at Marriage, and Children by Women's Education

	$\leq$ High School	Some College	College Ed.	Highly Ed.	Highly Ed. - College Ed.
Income	10,179	14,220	18,879	32,986	14,107***
Spousal Income	43,205	64,247	79,147	73,621	-5,526***
Age at marriage	21.28	22.42	23.63	24.36	0.73***
Children in HH	2.51	2.56	2.63	2.19	-0.45***
$\geq$ 4 children in HH	0.25	0.25	0.25	0.16	-0.09***

Notes: 1 percent 1970 Census data, women 41-50, except for children in household, which is measured for women 38-42, to avoid bias from children aging out of the household, weighted by Census person weights. Children ever born is available in 1970, but not in 2000 or 2010, which is required for the simulations. Additionally, it may not reflect true fertility, since in earlier years infant mortality was more prevalent. Nonetheless, the gap between College and Highly Educated women for this metric is extremely similar: 0.42 fewer children born, and a 0.11 lower chance of having $\geq$4 children.

utility matching model. In this model, human capital investments yield earnings gains, but can also delay marriage and childbearing, resulting in lower fertility. The dimensions of this model cannot be collapsed to an index, because fertility impacts the household's ability to create surplus through investing income in children, and thus its value is dependent on income.

Transferable utility matching models derive matching patterns from the efficient creation and division of surplus [Shapley and Shubik, 1971, Becker, 1973]. The equilibrium payoff of each individual is set in the market as "offers" where both spouses are able to attract one another. Thus, the model simply requires assumptions on the form of the marital surplus to establish equilibrium matching patterns and resulting utilities. As long as utility is fully transferable, the disaggregate equilibrium will be one and the same as the equilibrium that maximizes total social surplus.

Thus, to determine the stable match, we first must consider how household surplus is created.

3.0.1 Household Problem

Men are characterized by income, y, and women are characterized by both income, z, and fertility, p. p represents the probability of successfully conceiving. Individuals value private consumption, q, and children as a public good, Q, which are complementary, producing the underlying force toward assortative matching [Lam, 1988] .[8] With a single public and private good, the necessary and sufficient condition for transferable utility is generalized quasi-linear (GQL) utility [Bergstrom

[8] This can be thought of as the human tendency to want children to have similar levels of consumption as parents, a driving force in quantity-quality tradeoff models.

and Cornes, 1983, Chiappori and Gugl, 2014]. The simplest form of GQL is Cobb-Douglas utility, or "qQ" utility [Chiappori, 2017, Chiappori et al., 2017b], which I modify as $q(Q+1)$ so that the couple cares about private consumption even if children do not occur. Because utility is fully transferable, the allocation of income between children and private consumption can be found by maximizing the sum of utilities subject to the budget constraint. The impact of biological fecundity is captured by only allowing households to invest in Q if a child is born, with probability p. Assuming they have children, the couple's problem is thus:

$$\max_{q,Q} q(Q+1), \quad \text{s.t. } q+Q=y+z.$$

Accordingly, the utility maximizing level of q and Q are $q^*=\frac{y+z+1}{2}$ and $Q^*=\frac{y+z-1}{2}$.

If children were born with certainty, this would result in a very standard surplus function that is supermodular in incomes, and would thus predict assortative mating on the marriage market. However, households are constrained to $Q^{'}=0$ and $q^{'}=y+z$ in the case no children realize, with probability $1-p$. Thus, joint expected utility from marriage, T, is a weighted average between the optimal joint utility if a child is born and the constrained utility from allocating all income to private consumption: $T(y,z,p)=p\frac{(y+z+1)^2}{4}+(1-p)(y+z)$.

To find the surplus from marriage, we simply subtract out the utility from being single, which is to consume one's own income.[9] Thus, the marital surplus is $s(y,z,p)=p\frac{(y+z+1)^2}{4}+(1-p)(y+z)-y-z$, simplifying to:

$$s(y,z,p)=\frac{1}{4}p(y+z-1)^2. \tag{1}$$

3.0.2 Properties of Surplus Function

The surplus in 1 is supermodular in incomes, and also supermodular in income and fertility. For simplicity, this example uses a binary fertility outcome, but the model is easily extendable to a case with a set of possible family sizes, and a probability of achieving each one, while maintaining the same properties, as shown in Appendix B.4. This multiple children extension is the one I will use for simulations.

Because of supermodularity in incomes, for any two women of the same fertility level, matching will be positive assortative in incomes. However, when both income and fertility vary, whether

[9]Because the focus of this paper is educated women, and non-marital births make up an extremely small percentage of all births for women with greater than a college degree, I do not consider cohabitation or non-marital births. Future research may wish to examine the implications of reproductive capital in a model where cohabitation allows couples to capture a subset of the gains from marriages, such as in Calvo [2022].

the matching is positive or negative assortative on income can depend on the distribution of types, and in particular, the amount of fertility given up for an increase in income. In Appendix B.1.2, Proposition 3, I show that it is generically true for surpluses that are supermodular in both incomes and income and fertility that the stable match depends on the distribution of types.

However, because the complementarity between incomes *relative* to the complementarity between income and fertility in the surplus goes to zero as income goes to infinity, the stable match will always exhibit some section of negative assortative matching on incomes as long as the richest man is "rich enough." In Appendix Lemma 2, I show that in general, the necessary condition for this to be true is for the surplus to have the property that $\lim_{y\to\infty} \frac{\frac{\partial^2 s(y,z,p)}{\partial y \partial z}}{\frac{\partial^2 s(y,z,p)}{\partial y \partial p}} = 0$. The intuition for this is that the substitution between incomes in generating the household surplus, but not between husband's income and wife's fertility, and thus the marginal tradeoff between fertility and wife's income worsens as household income rises.

3.0.3 Distribution of Types

Women are divided into three types: low income and high fertility, L, medium income and high fertility, M, and high income and low fertility, H. This captures a key feature of biological fecundity, that it declines non-linearly past a certain age. As a result, some amount of human capital can be acquired without incurring reproductive capital losses, but larger human capital investments incur a reproductive capital penalty.[10] Roughly, you can think of the three types as being high school, college, and graduate-educated women.[11]

The three types of women have the following income–fertility pairs:

	z	p
L:	$\gamma - \mu_\gamma$	$\pi + \delta_\pi$
M:	γ	$\pi + \delta_\pi$
H:	$\gamma + \delta_\gamma$	π

[10] There may be other costs to education, in terms of foregone time or monetary costs. However, for this initial section, the education distribution is assumed to be exogenous. Additionally, it is possible that men make human capital investments as well, but these impact them uni-dimensionally.

[11] Appendix D shows that a model with continuous female skill produces highly similar predictions for aggregate matching patterns. Thus illustrating with three types does not limit the model's generality, but has the advantage of mapping well onto empirical exercises, where education is typically used as women's "type," since income is chosen endogenously post-marriage.

Thus, δ_γ is the income premium to being the high versus medium type, and δ_π is the fertility penalty. μ_γ is the income premium to being the medium versus low type. The mass of the three types of women is first assumed to be exogenously given, as g^K, where $K \in L$, M, H. Section 3.2 extends the model to allow for endogenous human capital investment.

There is a total measure 1 of women: $g^L + g^M + g^H = 1$. I assume there are more men than women, and thus only measure 1 of men can be matched.[12] Define the poorest man who receives a match as y_0 and the richest man as Y. Assume the income parameters are such that the poorest matched man's income plus the poorest woman's income is greater than 1 (this ensures interior solutions for the amount invested in children).

3.1 Matching Equilibrium

A matching is defined as the probabilities for each y for matching with each (z,p) type, and value functions $u(y)$ and $v(z,p)$ such that for each matched pair $u(y)+v(z,p)=s(y,z,p)$. A matching is stable if two conditions hold for all individuals:

$$u(y)+v(z,p) \geq s(y,z,p)$$
$$u(y) \geq y, \quad v(z,p) \geq z$$

That is, the utility received by any two individuals in their current matches must be jointly higher than the surplus they could create by matching together (the equation holds with equality if the pair is married to each other), and all individuals receive a positive benefit to marriage versus the outside option of consuming their own income.

The principle of surplus maximization allows us to think about the stable equilibrium in terms of maximizing the relative benefit of matching with different female types over men's income. The surplus benefits from changing types as a function of men's income are as follows. Medium versus low:

$$\Delta^{M-L}(y) = s(y,\gamma,\pi+\delta_\pi) - s(y,\gamma-\mu_\gamma,\pi+\delta_\pi) = \frac{1}{4}(\pi+\delta_\pi)\mu_\gamma(2y+2\gamma-\mu_\gamma-2).$$

High versus medium:

$$\Delta^{H-M}(y) = s(y,\gamma+\delta_\gamma,\pi) - s(y,\gamma,\pi+\delta_\pi) = \frac{1}{4}\pi\delta_\gamma(2y+2\gamma+\delta_\gamma-2) - \frac{1}{4}\delta_\pi(y+\gamma-1)^2.$$

[12] This is for simplicity in pinning down explicit utilities. There could be an equal number of men and women, or excess L-type women, with no change to the basic matching patterns.

With high versus low being the sum of the two.

$\Delta^{M-L}(y)$ is linear, and monotonically increasing in men's income. Thus, with fertility constant, there is always a higher surplus benefit from pairing a higher income man with a higher-income-type woman, corresponding to the supermodularity in the surplus function. Any stable match must therefore match M women with higher income men than L women.

Δ^{H-M} is quadratic, giving it a unique maximum, as is Δ^{H-L} . This quadratic form stems from the declining relative complementarity between incomes compared to income and fertility. Therefore, there is a single interval of men that it is maximally beneficial to pair with high-income women. However, this interval may not be the richest men. Note that even when $\Delta^{H-M}(y)$ is positive over the full range of y, indicating that there is a positive surplus benefit for all men to matching with H women over M women, the richest men may not be matched with the richest women, because they do not receive a sufficient benefit to pay the "price" these women command.

From this we can derive the following lemma.

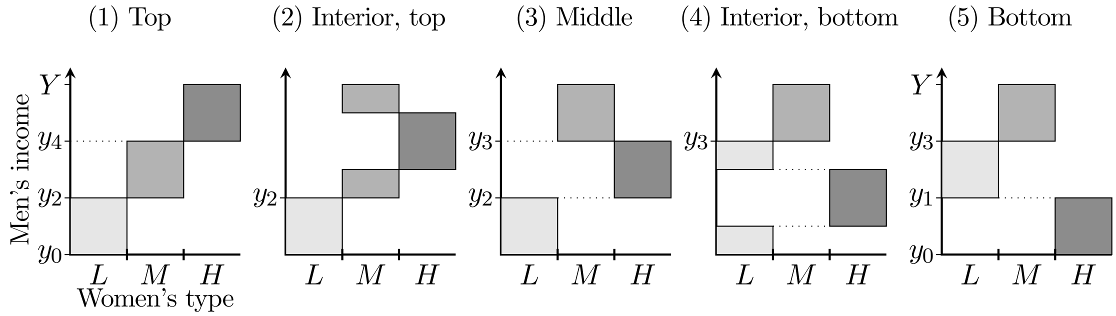
Lemma 1. *Any stable matching will exhibit the following three characteristics: (1) All matched men will be higher income than all unmatched men. (2) All men matched with M women must be higher income than all men matched with L women. (3) The set of men matched with H women must be connected.*

Proof. Item (1) follows from the fact that the surplus function is monotonically increasing in men's income (as long as the total household income exceeds 1, which was assumed). Item (2) follows from the fact that the benefit to matching with an M type versus an L type is monotonically increasing in income. Item (3) follows from the fact that the benefit of matching with an H type versus M or L type is single-peaked: If there is a gap in the men who are matched with H women, then the men in the gap must be matched with L or M women. But, as the benefit to matching with H women over L or M women is single-peaked, it cannot simultaneously be better to be matched with H women on both sides of the gap than in the gap. □

The options for the match that meet these criteria are illustrated in Figure 3, where the x-axis represents women's type and the y-axis represents men's income. The first picture illustrates standard assortative matching. All other possible match types meet the criteria in Lemma 3, and yet feature non-monotonicity in income-matching.

Type H women will be matched with the men who receive the most benefit from matching with them relative to type M or L women. Thus, we can think of characterizing the stable

Figure 3: Possible matches: H women match with...



equilibrium as sliding a segment of length h of men who match with type H women from the poorest man to the richest, stopping where the total surplus is maximized. If the man at the top of this segment benefits more than the man at the bottom, we should slide it up. If the man on the bottom benefits more than the man at the top, we should slide it down.

Thus, assortative matching will only be stable when the man with income Y receives more benefit to an H match than the poorest man to be matched with an H type, labeled as y_4 in Figure 3.[13] We can thus create a condition for positive assortative matching using Δ^{H-M}, and derive the following proposition.

Proposition 1. *Let Y represent the income of the richest man. For any set of parameters, it is possible to find a Y large enough such that the equilibrium match is non-monotonic in income.*

Proof. Assortative matching requires that $\Delta^{H-M}(Y) \geq \Delta^{H-M}(y_4)$, because otherwise the total surplus can be increased by matching the man right below y_4 with an H type woman, and Y with an M type woman. This condition reduces to: $\frac{\pi}{\delta_\pi}\delta_\gamma \geq \frac{1}{2}(Y+y_4)+\gamma-1$, which relies linearly on Y. Assume this condition is met. Increasing Y sufficiently will cause the condition to be violated, in which case matching Y with an H-type woman cannot be surplus maximizing. $\square$

We can envision moving through each of the equilibria by slowly increasing Y relative to the other parameters. When the condition for assortative mating fails, H-type women match interior to the segment of men matching with M-type women, as shown in equilibrium 2, such that the benefit of the bottom man matching with an H-type with income y^* is exactly equal to the income of the top man with income y^{**}: $\Delta^{H-M}(y^*) = \Delta^{H-M}(y^{**})$.[14] As Y increases, the

[13] These thresholds have specific definitions in terms of the distributions, but I name them for notational simplicity. $y_4 = F^{-1}(1-g^H)$, $y_3 = F^{-1}(1-g^M)$, $y_2 = F^{-1}(g^L)$, and $y_1 = F^{-1}(g^H)$.

[14] where $y^{**} = F^{-1}(F(y^*)+g^H)$.

segment of men matching with H women will continue sliding down until there are no more M women. At that point, equilibrium 3, where H women are matched exactly between L and M women, will be stable as long as the last man matched with an M woman, with income y_3, receives a higher benefit from an H versus L match than the richest man matched with an L-type woman, y_2. If this condition is violated, the segment of men matching with H women will slide down further, interior to the men matching with L-type women, as in equilibrium 4, such that $\Delta^{H-L}(y^*) = \Delta^{H-L}(y^{**})$. Finally, if Y continues to increase, at some point the lowest income man, y_0, will be matched with an H type woman, as in equilibrium 5.

A full characterization of the equilibrium, and a single-variable maximization problem to determine the stable match for any set of parameters, is shown in Appendix B.2. Appendix Figure A5 illustrates the conditions on the surplus differences to maintain each equilibrium, with men's income changing relative to fixed parameters. The equilibrium will also change if the distribution between income and fertility in female types changes. Figure A4 shows the range of δ_γ, the financial return to investment, and δ_π, the fertility penalty, that support different equilibria types. As δ_γ/δ_π increases, the matching progresses from equilibrium 5 to equilibrium 1, assortative matching.

3.1.1 Utility

Transferable utility matching models allow the direct calculation of each individual's equilibrium utility (value function). This is done through using the equilibrium stability condition that $u(y)+v(z,p) \geq s(y,z,p)$, and that marriages improve welfare over singlehood. This procedure is shown in Appendix B.3. Let women's value functions for each type be denoted U^K, and the marital surplus each type receives as v^K, constants that depend on the underlying parameters, including δ_π. Then women's equilibrium utility for each type will be:

$$
\begin{aligned}
U^H &= \gamma + \delta_\gamma + v^H, \\
U^M &= \gamma + v^M, \\
U^L &= \gamma - \delta_\mu + v^L.
\end{aligned}
$$

Note that individuals maximize the surplus they receive, rather than the "quality" of their partner. So, from the woman's perspective, a non-assortative equilibrium can be viewed as women choosing relationships in which they have more "bargaining power," thus receiving a larger share of a slightly smaller pie. They can command this higher surplus share when they create more relative value in relationships with lower earning partners.

However, the H-type equilibrium utility function is affected by the fertility loss associated with education, both through her own lower utility from children, and through the equilibrium channel of a smaller marital surplus share.

3.2 Endogenous Human Capital Investment

Both the personal and marriage market impacts of human capital investment will influence women's willingness to invest in human capital in the first place. The reproductive capital loss creates an extra "tax" on women's human capital investments, reducing the returns to intensive human capital investments. However, it is still possible to sustain an equilibrium where women invest in costly human capital, even if in doing so they forego the most favorable marriage market matches.

Assume that the distribution of L types is fixed, but that M types can invest to become H types. Further assume that women considering investing face a utility cost, c_i, of investment. Using the equilibrium value functions, women will invest in becoming the high type when:

$$c_i \leq U^H - U^M$$
$$c_i \leq v^H - v^M + \delta_\gamma$$

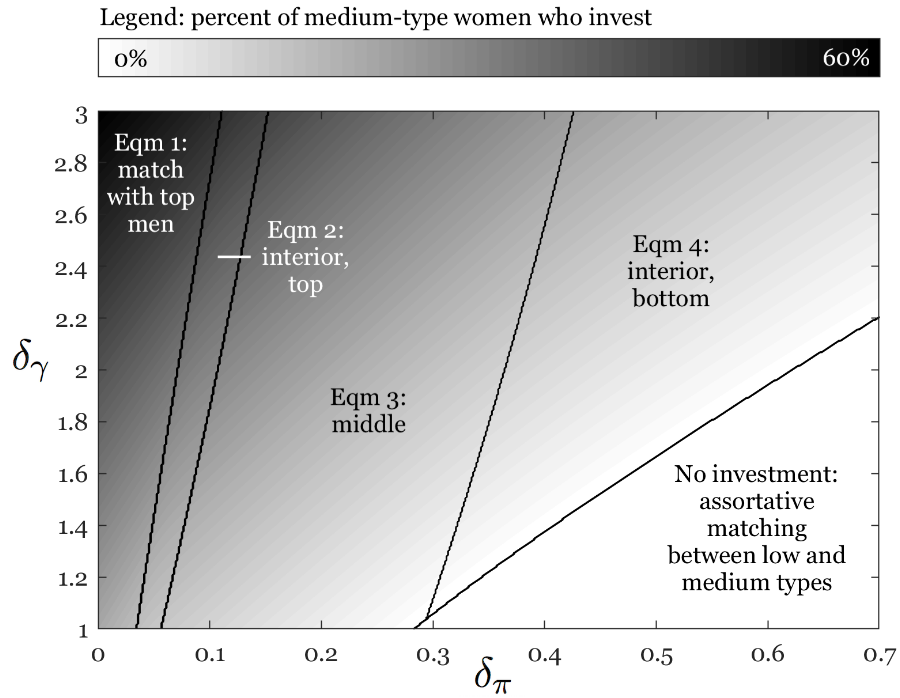
The mass of H types will now be endogenously determined as a function of the underlying density of c_i. This mass affects the marital surpluses for each female type through its impact on the y at the boundary between different wife types. Thus, the cutoffs for women investing can be solved for as a fixed point of $c_i = v^H(c_i) - v^M(c_i) + \delta_\gamma$. Call the solution to this equation $\hat{c}$. There will be a unique equilibrium where all women with costs below $\hat{c}$ invest in becoming the H type, and then match according to Proposition 1. If no women invest, the matching will be assortative between L and M types. The threshold cost for investment $\hat{c}$ is decreasing in δ_π (fewer women invest as the fertility cost rises) and increasing in δ_γ (more women invest as the income premium rises).

Figure 4 illustrates the portion of women that invest and resulting matching equilibria for different income gains and fertility costs of investments, and a heterogenous uniform utility cost.[15] Importantly, some women invest in all possible marriage market equilibria, except when H-type women are matched with the absolute lowest income men.

The figure illustrates the interesting difference in the forces driving women's investment decision

[15]The thresholds for the matching equilibria are somewhat different than in Appendix Figure A4, as the equilibrium responds endogenously to the number of educated women on the market.

Figure 4: Matching Equilibrium and Investment by Income Return and Fertility Penalty



Notes: Figure illustrates the δ_γ and δ_π space that supports each matching and investment equilibrium. Bounds calculated with men's income uniform from 0 to 6, and for women M-type income of 4, L-type income of 2, with a mass of 0.35 L types, and 0.65 M types who have the option to invest. Baseline fertility is 0.3. Cost of investment ranges uniformly from 0 to 12.

versus the marriage market equilibrium. Women's investment changes more in δ_γ, the financial return to investment, while the marriage market equilibrium is more influenced by δ_π the fertility penalty. This is because women get the direct financial benefit of their investment in addition to the marriage market payoff, and thus receive an extra financial incentive to invest that does not appear in the marital surplus, which is what influences the matching equilibrium. Thus, women may still be better off investing than not, and yet experience a lower match quality.

3.3 Predictions

The model predicts that the matching relationship between men's and women's income can be non-monotonic. If income and fertility are negatively related in the distribution of women, the richest men will not be matched with the richest women as long as either the fertility-income tradeoff is large enough or the income of the richest man is high enough. This prediction matches the stylized facts shown in Section 2.

The model additionally predicts that if either δ_π, the fertility penalty to investment, falls, or δ_γ, the income premium, rises, matching will become more assortative at the top of the female income distribution, and women will invest more in human capital. An increase in δ_γ over time is likely as the skill premium rises and women experience fewer barriers in high-earning professions. And δ_π is likely to be falling over time, both due to improved technology and falling desired family sizes.

Finally, the model could also be extended to predict higher marriage and lower divorce rates for highly educated women over time, by using a stochastic joint shock that makes some couples choose not to marry, and others to divorce. Higher surplus couples would be more resilient to higher shocks, and so marriage would be proportional to the surplus of the couple, with the surplus in couples with high-income women rising as matching becomes more assortative and they match with richer men.

4 Empirical Relevance

In this section, I demonstrate the relevance of a bi-dimensional model with "reproductive capital" to historical data by simulating the model with moments from Census data, and demonstrating it can match shifts in matching patterns and marriage rates over time. Note, although this simulation exercise is quite simple, its goal is to demonstrate that a bidimensional model where education affects income and fertility can be useful in explaining observed patterns in marriage market matching. By replicating the shift in matching patterns over the last 50 years, this section shows that the phenomenon of increased assortative mating at the top of the distribution can be explained only by shifts in highly educated women's relative fertility and income, without requiring broader changes in social norms.

To make the simulation more realistic, this section allows families to have multiple children by introducing a slightly more complex structure than a single fertility probability. Instead of constraining investments in children to be zero with probability p, I use an education-specific probability of achieving each family size, c, and constrain investments in children proportionally based on realized family size, resulting in the following surplus (see Appendix B.4 for details on how this surplus is produced from the underlying household problem):

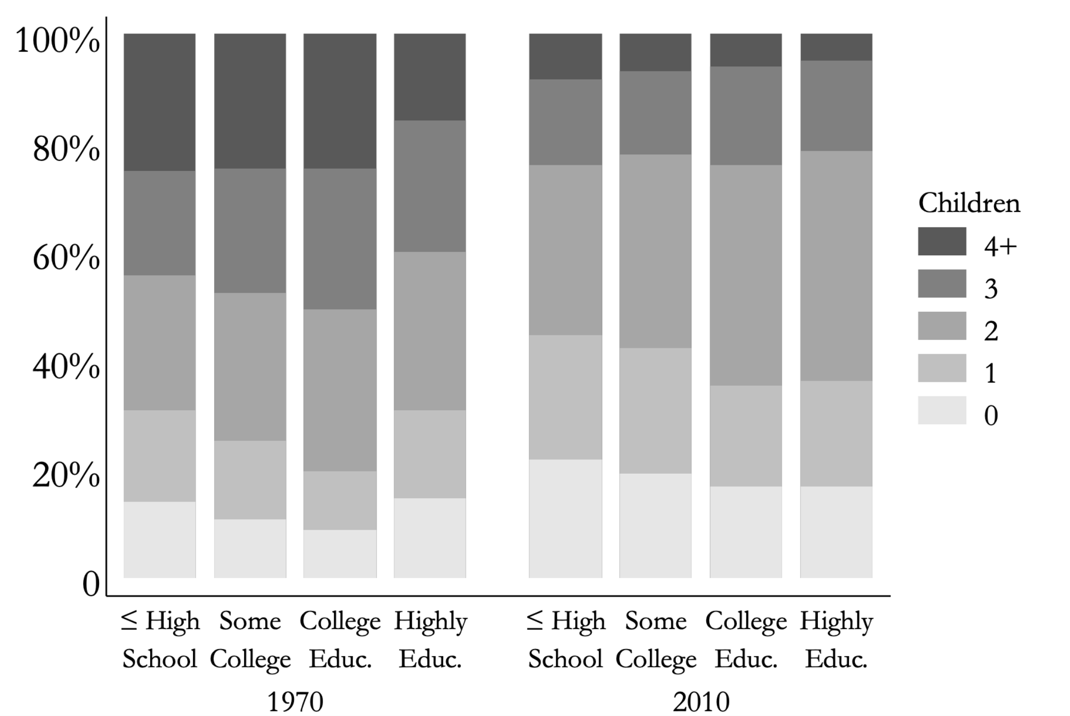
$$s(y,\, z,\, p) = \sum_{c=0}^{4} \left[p_c \left(y + z - \frac{c}{4} \frac{y+z+1}{2} \right) \left(\frac{c}{4} \frac{y+z-1}{2} \right) \right] - y - z \tag{2}$$

4.1 Data Moments

Two key pieces of data from the decadal Census are used to feed the simulation. First, incomes of men and women conditional on education. And, second, an education-specific fertility distribution. For income, men's income is drawn unconditional on education, and then a net present value is calculated to approximate lifetime earnings. The patterns shown are not sensitive to the exact method of approximation. Women's income is drawn conditional on educational level, for women working full-time, proxying the earning potential of a given education level, since couples may decide to reallocate women's human capital toward home production. Details are provided in Appendix C.1.

For fertility, I use the empirical distribution of number of children conditional on education. As shown in Figure 5, in 1970 highly educated women had substantially fewer children and a higher probability of having zero children than college educated women, while college educated women do not have a lower chance of having children than those with lower education levels. Highly educated women marry approximately one year older than college educated women on average, but the fertility difference likely also stems from longer child spacing and a higher opportunity cost of maternity leave. Thus, I take the overall lower fertility as the best measure of the tradeoff in reproductive capital from achieving higher income.

Figure 5: Empirical Distribution of Children by Education Level and Year



Notes: Children currently at home for women 38-42 years old. Children ever born, only available through 1990, produces qualitatively similar results, in Appendix Figure A6. "Highly educated" constitutes all graduate degrees. Source: 1 percent Census data from 1970 and American Community Survey from 2010, weighted by person weights.

These moments change over time, which will drive changes in matching patterns in the model. Market opportunities for women have naturally risen dramatically in the past 50 years [Hsieh et al., 2019]. However, a more dramatic shift might come from changes in the fertility penalty associated with investment. One reason is changing technology, including fertility drugs, in vitro fertilization, surrogacy, egg donation, and egg freezing (e.g., Gershoni and Low [2021a,b]). Perhaps more importantly, a trend toward smaller family sizes [Doepke and Tertilt, 2009, Gould et al., 2008, Isen and Stevenson, 2010, Preston and Hartnett, 2010] causes college educated women's fertility to be more comparable to highly educated women's fertility, as shown in Figure 5, where college and graduate-educated women have a nearly identical fertility distribution by 2010. This change could be driven by an increasing preference for child quality over child quantity, which tends to accompany economic development [Becker et al., 1990].[16]

4.2 Simulation Compared to Data

Figure 6 shows on the left-hand side the actual changes in spousal income for women of different education levels over time, from Census data. The right-hand side simulates these same changes using hypothetical spouses from matching according to surplus maximization of the function in equation 2, plugging in the changes in the income distribution of men and women and the empirical distribution of children over time.

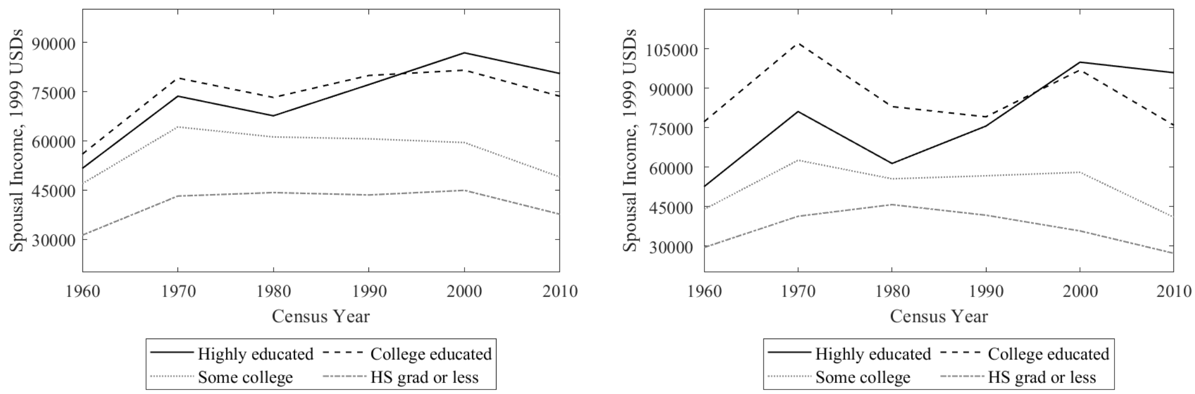
Using only changes in income and fertility over time, the simulation captures the crossing between the spousal incomes of college-educated and graduate-educated women, matching the timing between the 1990 and 2000 Censuses. The matching patterns between other groups remain stable in the simulation, as they do in the data. While the simulation somewhat over-estimates the differences between groups, as individuals are assumed to match on income and fertility only, it demonstrates that even a very simple model incorporating fertility can explain the transition from a non-monotonic relationship between spousal income and women's education to assortative matching.

Figure 7 compares the actual path of women's investment in graduate education to that predicted by the model. The distribution of the cost of education, c_i, is calibrated to match the rates of graduation in 1970. Using that same cost distribution, the model matches the increase in 1980 and 1990, although slightly overestimates the increase in 2000 and 2010. Note, the distribution of c_i used in the simulation is uniform–a cost distribution with lighter tails may do a better job matching changes in later years. Nonetheless, this demonstrates that even a simulation of the model without extensive calibration can rationalize historical facts.

[16] Not only did actual family sizes fall, but *desired* family sizes have fallen substantially. During the 1970s, there was a rapid transition from "four or more" as the modal answer for ideal family size to "two," shown in Appendix Figure A7 [Livingston et al., 2010].

Figure 6: Spousal Income by Education Group

(a) Data (b) Simulation



Notes: Left-hand side figure comes from 1% Census data from 1960, 1970, 1980, and 1990, and American Community Survey from 2000 and 2010, women 41-50 years old, weighted by Census person weights. Right-hand side is model simulation taking random weighted draws of income (conditional on education for women), and then calculating an NPV of approximate lifetime income conditional. Fertility distribution is for women 38-42, so children are still at home. Income is translated into an NPV of 25 periods for men and 20 for women (robust to other choices). Using these inputs, matches are determined to maximize total surplus, and then average income of predicted spouse is graphed by education group.

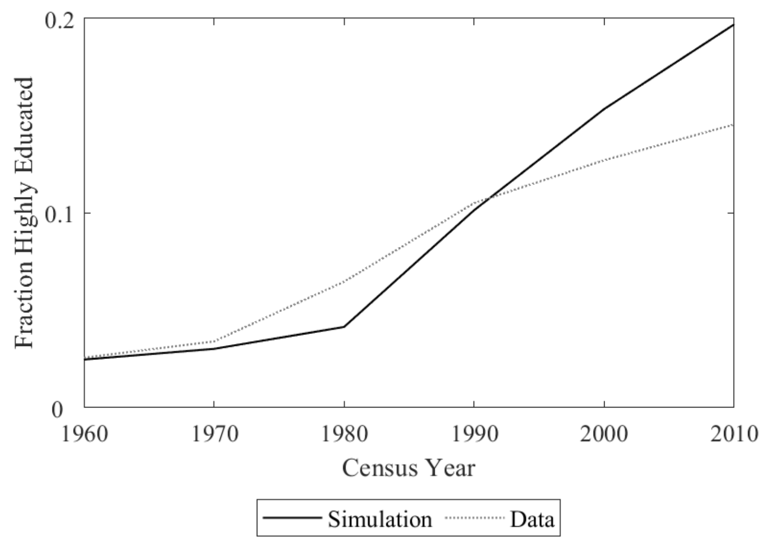
When the spousal matching patterns are re-estimated using endogenous education decisions, matching patterns are similar, although it exaggerates the difference between highly and college educated women's spousal income in later years because of the additional selection effect, shown in Appendix Figure A8. The model can also produce a simulation of marriage rates that matches the increasing rates of marriage for highly educated women relative to college educated women over time, shown in Appendix Figure A9.

This exercise demonstrates that a model where women's reproductive and human capital both matter in matching can reconcile historical patterns in women's education decisions and marriage market outcomes. The model's bi-dimensionality is key to its success in matching the increasing spousal income at most levels of education—those that do not carry significant fertility penalties—and decreasing spousal income, until recently, between college and graduate education.

5 Conclusion

This paper treats women's decisions as a tradeoff between two assets: human capital, which grows based on investment, and reproductive capital, which depreciates with time. I develop a bi-dimensional marriage matching model where women's career investments affect both human and reproductive capital. Matching is predicted to be non-monotonic when the fertility cost of career investments are large relative to the income gains. This adds a second cost to women considering

Figure 7: Predicted Education



Notes: Model simulation of endogenous education decision, with uniform education cost, taking NPV of approximate lifetime income conditional on education and fertility as inputs. Matching and education decisions are determined to maximize surplus, as the private education decision will match the efficient equilibrium.

time-consuming career investments—not only do they themselves potentially lose out on fertility, but they experience a "tax" on the marriage market as well. I document in US Census data that until recently marriage matching followed the non-monotonic pattern predicted by the model. As family size desires fell and reproductive technology expanded, equalizing family sizes between college and graduate educated women, there has been a transition to more assortative mating, higher rates of graduate education, and higher marriage and lower divorce rates for graduate educated women. These patterns are matched by a simulation of the model.

This paper provides an example where a bi-dimensional matching framework is crucial to rationalizing surprising patterns in the data. Moreover, it explores the occurrence of non-monotonicty in income matching as a potentially general feature of such bi-dimensional models where one side's traits are negatively correlated. Thus, reproductive capital may be an important consideration in understanding both marriage patterns and women's human capital decisions, with the fertility costs of human capital investments impacting women economically, whether or not they desire children themselves.

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Online Appendix

A Appendix: Stylized Facts

Figure A1 shows that the decline in spousal income with age at marriage holds for women currently age 36-45, married up to age 35. Table A1 demonstrates that the difference in spousal income between college and highly educated women is statistically significant. Figure A2 shows that the non-monotonic relationship is also present when restricting to first marriages only, omitting 1990 and 2000 when number of marriages is not available.

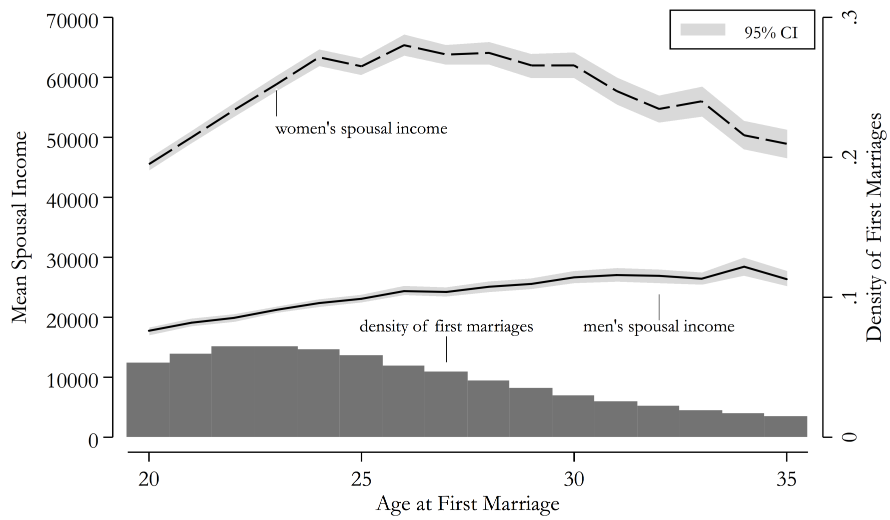
Table A1: Spousal income by wife's education level

	Dependent variable: Spousal income, 1999 USD		
	(1)	(2)	(3)
1960 × highly ed.	-3,619***	-3,435***	-3,394***
	(1,182)	(1,181)	(1,181)
1970 × highly ed.	-5,947***	-5,788***	-5,740***
	(1,341)	(1,341)	(1,341)
1980 × highly ed.	-6,081***	-6,088***	-6,093***
	(893.1)	(893.9)	(893.8)
1990 × highly ed.	-3,164***	-3,197***	-3,185***
	(1,037)	(1,036)	(1,037)
2000 × highly ed.	4,320***	4,278***	4,305***
	(1,029)	(1,033)	(1,034)
2010 × highly ed.	7,098***	7,093***	7,112***
	(882.3)	(882.4)	(882.4)
Constant	55,406***	52,623***	58,503***
	(645.8)	(2,131)	(3,261)
Year FE	Y	Y	Y
YOB FE		Y	Y
Spouse age			Y
Observations	118,538	118,538	118,538

Notes: Regressions of spousal income on wife's education level interacted with year for women with at least a college degree, with "highly educated" constituting all formal education beyond a college degree. No constant or "highly" dummy is included, so coefficients can be interpreted as the additional spousal income for those in the highly educated category in each sample. Source: 1 percent samples of US Census data from 1960, 1970, 1980, 1990 Census, and 2000 and 2010 ACS, married women 41-50 years old, weighted by Census person weights. Robust standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.1

Figure A1: Spousal Income by Age at Marriage, Before Age 35



Notes: Lines represent the average spousal income for first marriages by age at marriage for women versus men. Bars represent the portion of all women's marriages occurring at that age, to check whether selection is driving the effect. Source: 2010 American Community Survey (1 percent sample) marital histories for white men and women, 36-45 years old.

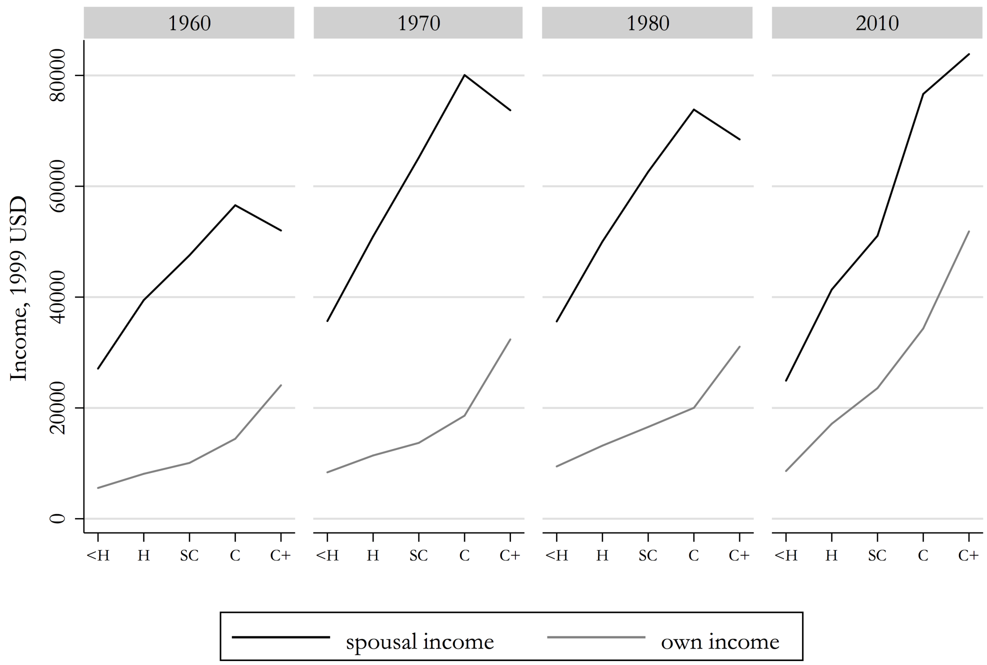
Marriage and divorce rates Highly educated women's marriage *rates* have also risen precipitously, shown in Figure A3, another measure of an increasing marriage market premium for highly educated women. Conventional wisdom holds that too-educated, too-high-earning women are punished on the marriage market. However, Figure A3 panel (a) demonstrates that *college* educated women actually always married at rates close to all other educational categories. It is only *highly* educated women who previously had comparatively low rates of marriage, and have now experienced substantial gains. Similarly, as shown in panel (b), highly educated women previously divorced at the highest rates, while college educated women's divorce rates were on par with other educational categories. Since 1990, highly educated women's divorce rates have fallen while college educated divorce rates have leveled off, and all other categories' have risen. These trends are both matched by the model simulation.

B Appendix: Model

B.1 General Model

This section demonstrates that a simple bi-dimensional model where couples care about income and fertility can produce non-monotonic matching in incomes. In particular, matching will always

Figure A2: Non-monotonicity in spousal income by wife's education level, First Marriages Only



Notes: Income of spouse based on wife's education level. <H=less than high school, H=high school grad, SC=some college, C=college grad, C+=graduate degree. Source: 1 percent Census data from 1960, 1970, and 1980, and ACS data from 2010. Sample consists of women, ages 41-50 years old, who are on their first marriage.

be assortative when women's income varies, but fertility stays constant, but need not be when income and fertility vary in tandem.

B.1.1 Model Setup

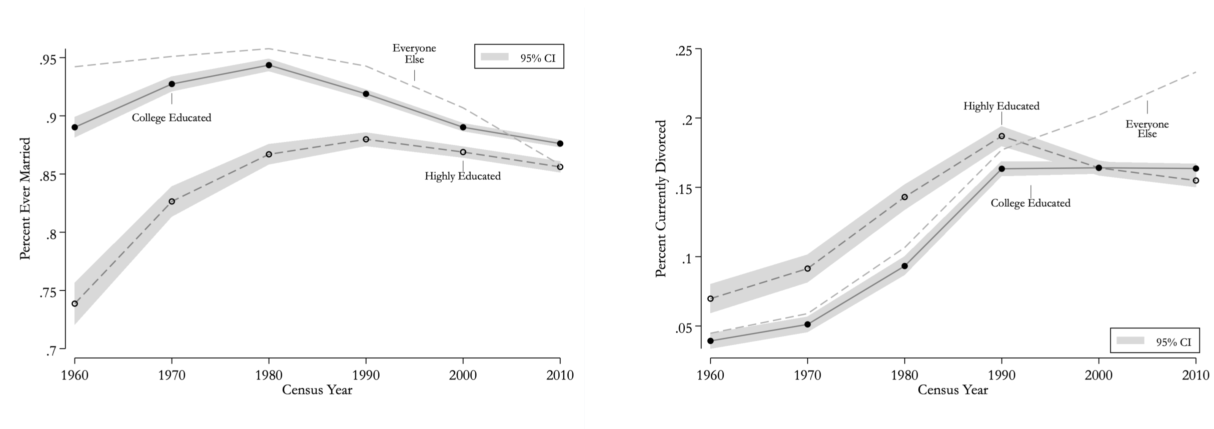
There is a two sided market with a unidimensional side, "men," and a bi-dimensional side, "women." Men are characterized solely by income, $y \in [0,Y]$, and women are characterized by both income and fertility, $(z,\, p) \in [0,Z] \times [0,1]$.[17]

Individuals care about both children and private consumption, with utility meeting the generalized quasi-linear (GQL) form, which is the necessary and sufficient condition for transferable utility [Bergstrom and Cornes, 1983, Chiappori and Gugl, 2014]. Thus, through maximizing the sum of utility we can identify the household surplus function, $s(y,\, z,\, p)$, increasing in all arguments.

[17] Although making men unidimensional is a simplification, it should be noted that other matching models that feature multiple characteristics are actually unidimensional as long as the characteristics can be collapsed to a single index. I focus on income as that is the key factor usually examined in models looking at societal trends in assortative matching. Many of the predictions here would also hold for other factors that could be part of a quality index, such as height or attractiveness.

Figure A3: Marriage and divorce rates by education level

(a) Marriage (b) Divorce



Notes: Ever married and currently divorced rates by ages 41-50, for women, based on education level, with "highly educated" constituting all formal education beyond a college degree. Ever divorced rates show a similar pattern, but are not available in all years. Source: 1 percent Census data from 1960, 1970, 1980, and 1990, and American Community Survey from 2000 and 2010. Sample consists of women, ages 41-50 years old, weighted by Census person weights.

Let the surplus exhibit: (1) Supermodularity in incomes: $\frac{\partial s^2}{\partial y \partial z} > 0$, and (2) Supermodularity between men's income and fertility: $\frac{\partial s^2}{\partial y \partial p} > 0$.

B.1.2 Matching Equilibrium

This surplus function will always produce assortative matching in incomes when fertility is equal across women.

Proposition 2. *Let there be two men, y and y', with $y < y'$. For any two women with incomes z and z', $z' > z$, and fertility p, the stable matching matches y' with (z', p) and y with (z, p).*

Proof. With transferable utility, the stable match will maximize total surplus. Thus, supposing by contradiction that y is paired with z' and y' with z, it must be that:

$$s(y,\, z',\, p) + s(y',\, z,\, p) > s(y',\, z',\, p) + s(y,\, z,\, p)$$

$$s(y,\, z',\, p) - s(y,\, z,\, p)) > s(y',\, z',\, p) - s(y',\, z,\, p)$$

Which would imply that, for a small change in z, $\frac{\partial s(y,\, z,\, p)}{\partial z} > \frac{\partial s(y',\, z,\, p)}{\partial z}$, which would mean s is submodular, contradicting the premise. $\square$

However, when fertility differs, the matching can be positive assortative or negative assortative on incomes, depending on the income-fertility tradeoff for women.

Proposition 3. *Let there be two women* (z, p) *and* (z', p') *with* $z<z'$, $p>p'$. *Let* $\epsilon=z'-z$ *and* $\eta=p-p'$, *both positive. Let* $\lambda=\frac{\epsilon}{\eta}$. *There exists a* λ *such that the stable matching matches* y *with* (z, p) *and* y' *with* (z', p'), *and a smaller* λ *such that the stable matching matches* y *with* (z', p') *and* y' *with* (z, p).

Proof. By total surplus maximization, whenever $s(y, z, p)+s(y', z', p')>s(y, z', p')+s(y', z, p)$, y will be matched with (z, p) and y' with (z', p'), whereas when the opposite holds, y will be matched with (z', p') and y' with (z, p).

Define:

$$\begin{aligned}\bar{\Delta}&=s(y, z, p)+s(y', z', p')-s(y, z', p')-s(y', z, p)\\&=s(y, z, p)-s(y, z', p')-(s(y', z, p)-s(y', z', p'))\end{aligned}$$

For ϵ positive and small enough, the sign is the same as:

$$\begin{aligned}\bar{\Delta}&=-\frac{\partial s(y, z, p)}{\partial z}\epsilon+\frac{\partial s(y, z, p)}{\partial p}\eta-\left(-\frac{\partial s(y', z, p)}{\partial z}\epsilon+\frac{\partial s(y', z, p)}{\partial p}\eta\right)\\&=\left[-\lambda\frac{\partial s(y, z, p)}{\partial z}+\frac{\partial s(y, z, p)}{\partial p}-\left(-\lambda\frac{\partial s(y', z, p)}{\partial z}+\frac{\partial s(y', z, p)}{\partial p}\right)\right]\eta.\end{aligned}$$

Then:

$$\bar{\Delta}=\int_{y}^{y'}\left(\lambda\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial z}-\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial p}\right)\eta\; d\theta$$

Now, define:

$$\begin{aligned}m_z&=\min_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial z},\; M_z=\max_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial z},\\m_p&=\min_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial p},\; M_p=\max_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial p}.\end{aligned}$$

These are the maximum and minimum complementarities between incomes and income and fertility exhibited over the domain of types. Under the assumptions made, $m_z > 0$ and $m_p > 0$. Then:

- for $\lambda \geq M_p/m_z$, $\bar{\Delta} > 0$, and the first pattern obtains.
- for $\lambda \leq m_p/M_z$, $\bar{\Delta} < 0$, and the second pattern obtains.

$\square$

The intuition for the proof is that the stable match must maximize surplus, and because the surplus is supermodular between both men's income and women's income and fertility, it is always possible for the supermodularity in income-fertility to "outweigh" that in incomes for a sufficiently high quantity of income relative to fertility. In other words, the form of the stable match depends on the distribution of women's traits.

These two propositions together imply non-monotonic matching is possible when some women differ only in income, while others differ in income and fertility.

Lemma 1. *Let z(y) represent the income of the woman matched with man of income y. In a distribution where men vary in income, and women vary in both income and fertility, where fertility is always weakly decreasing in income, it is possible for there to be two men with incomes y and $y' > y$ such that $z(y') > z(y)$, and a third man with income $y'' > y'$ where $z(y'') < z(y')$.*

Proof. From the proof of Proposition 1, we know that if two women have the same fertility and different income levels, there must be assortative matching. Furthermore we know it is possible to have matching be negative assortative on incomes when women have different fertility levels, for λ low enough. Thus, to have non-monotonic matching simply requires that the the distribution has three women (z, p), (z', p), and (z", p') where $z'' > z' > z$ and $p' < p$ such that s(y", z', p) + s(y', z", p') ¿ s(y", z", p') - s(y', z', p), which we know will be true for λ high enough. Note, it is also possible to have assortative matching for the first two women if fertility differs, as long as λ is sufficiently low. $\square$

If the surplus further meets the condition that the relative complementarity of income compared to fertility with men's income goes to zero as men's income increases, then we can guarantee that there will always be a man rich enough such that he matches non-assortatively in the stable match.

Lemma 2. *For $s(y,\, z,\, p)$ such that $\lim_{y\to\infty}\frac{\frac{\partial^2 s(y,z,p)}{\partial y\partial z}}{\frac{\partial^2 s(y,z,p)}{\partial y\partial p}}=0$, for any λ there exists a Y large enough that the stable match does not match him with the highest-income woman.*

Proof. Assume by contradiction assortative matching everywhere. Then the richest man Y is matched with the richest woman, with income and fertility $(z',\, p')$. Define man $\bar{y}$ that is ϵ below Y and matched with woman with income and fertility $(\bar{z},\, \bar{p})$, where $z' > \bar{z}$ and $p' < \bar{p}$. Then it must be that $s(Y,\, z',\, p') + s(\bar{y},\, \bar{z},\, \bar{p}) > s(Y,\, \bar{z},\, \bar{p}) + s(\bar{y},\, z',\, p')$. Rearranging, the left-hand side becomes $s(\bar{y},\, \bar{z},\, \bar{p}) - s(\bar{y},\, z',\, p') - (s(Y,\, \bar{z},\, \bar{p}) - s(Y,\, z',\, p'))$, which has the same sign (see Appendix B.1 for details) as:

$$\int_{\bar{y}}^{Y}\left(\lambda\frac{\partial^2 s(\theta,\, z,\, p)}{\partial\theta\partial z} - \frac{\partial^2 s(\theta,\, z,\, p)}{\partial\theta\partial p}\right)\eta\; d\theta.$$

For Y high enough, this will always be negative, since λ is fixed and $\frac{\frac{\partial^2 s(y,z,p)}{\partial y\partial z}}{\frac{\partial^2 s(y,z,p)}{\partial y\partial p}}$ is positive and goes to zero as y increases. Thus the richest man cannot be matched with the richest woman for Y high enough. □

Thus, a matching model that is unidimensional on one side and bi-dimensional on the other can easily match empirical patterns exhibiting non-monotonicty along the “main” trait. In fact, non-monotonicity will be a general feature of the model when the women’s traits are negatively correlated and the top of the male income distribution is sufficiently high. Such bi-dimensional matching frameworks may provide a way to reconcile the general tendency toward assortative matching with deviations that appear to suggest some people do not value income. Rather, it is likely that income is negatively correlated with another valuable trait.

B.1.3 Generalizing to Other Settings

While this model is described in terms of income and fertility, its insights can be generalized more broadly. First, in terms of marriage market matching, one can think about insights for any situation in which one side has two salient traits that are negatively correlated and whose contribution to the surplus increases in the other side’s quality. This is in contrast to other useful work on bidimensional marriage models, such as Coles and Francesconi [2019], where the value of partner income is separable from own income. The key insight of this work is that even with supermodularity in incomes, it is possible to have non-assortative matching depending on the underlying distribution of traits and the change in relative complementarities across the income distribution. This provides predictions in line with the empirical regularity of largely positive assortative matching on income.

The model's insights can also be applicable to settings outside of fertility where one side of the market is bi-dimensional, and the value of both traits is increasing in the other side's characteristics. For example, manufacturers defined by quality and matching with upstream suppliers may care about both quality and speed. If these supplier traits are negatively correlated, non-monotonic matching in qualities is possible depending on the distribution of types. If furthermore the complementarity between qualities relative to the complementarity between quality and speed is decreasing in own quality, the model produces the result that for any distribution of quality and speed among the suppliers, a sufficiently high quality manufacturer will not be matched with the highest quality supplier. This type of model may help explain non-assortative matching in a variety of markets where one expects supermodularity in the most salient trait.

B.2 Parameterized Model Equilibrium

Proposition 4. *The unique stable match is fully characterized by Lemma 1 and the following conditions:*

- *If* $\Delta^{H-L}(y_1) \leq \Delta^{H-L}(y_0)$,

  H women match with poorest men, from y_0 *to* y_1.

- *If* $\Delta^{H-L}(y_3) < \Delta^{H-L}(y_2)$ *and* $\Delta^{H-L}(y_1) > \Delta^{H-L}(y_0)$,

  H women match with men interior to the set matching with L women, where $\Delta^{H-L}(\underline{y}^*) = \Delta^{H-L}(\underline{y}^* + h)$.

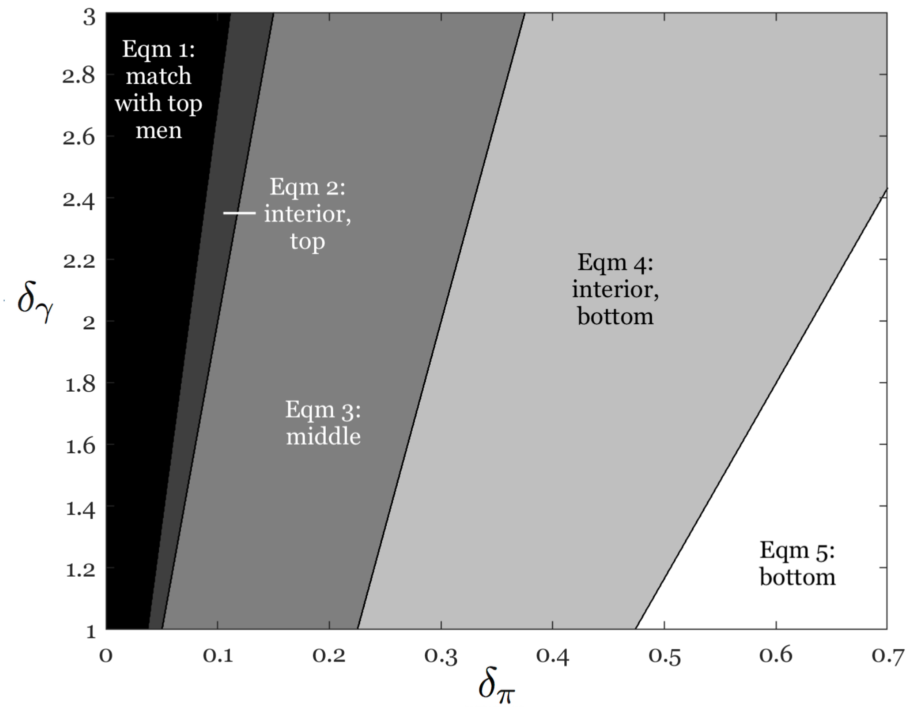
- *If* $\Delta^{H-L}(y_3) \geq \Delta^{H-L}(y_2)$ *and* $\Delta^{H-M}(y_3) \leq \Delta^{H-M}(y_2)$,

  H women match with middle men, from y_2 *to* y_3.

- *If* $\Delta^{H-M}(Y) < \Delta^{H-M}(y_4)$ *and* $\Delta^{H-M}(y_3) > \Delta^{H-M}(y_2)$,

  H women match with men interior to the set matching with M women, where $\Delta^{H-M}(\underline{y}^*) = \Delta^{H-M}(\underline{y}^* + h)$.

Figure A4: Matching equilibrium by return to investment and fertility penalty



Notes: Men's income ranges uniformly from 0 to 6 (total mass of 1), and for women there is a mass of 0.35 L types, and 0.35 M types, and 0.3 H types. M-type income is 4, L-type is 2. Baseline fertility is a 0.3 chance of conceiving.

- *If* $\Delta^{H-M}(Y) \geq \Delta^{H-M}(y_4)$,

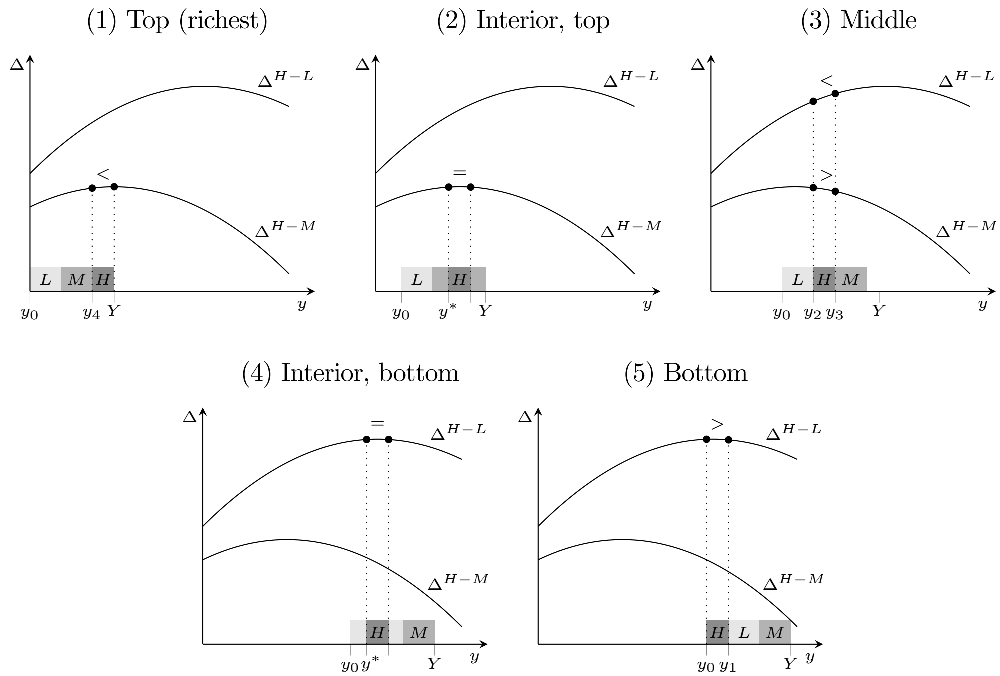
  H women match with richest men, from y_4 to Y.

Proof. The conditions in Lemma 1 create a single-variable maximization problem that has a unique solution for any given parameters, as shown in Appendix B.2. The solution is found through the first-order conditions of the problem, and the cutoffs for each equilibrium type is found through the boundaries for corner solutions. □

These conditions are illustrated in Figure A5. Note that although this figure illustrates the changing equilibria by keeping other parameters fixed and moving the range of y, it is also possible to keep men's distribution fixed and move the distribution of women's types, through changing the human capital return to investment or its fertility penalty, as is likely with technological change.

The matching equilibrium for any set of parameters can be solved for by a single variable optimization problem. Recall the mass of each female type is g^K, where $g^L + g^M + g^H = 1$, and men from y_0 to Y are matched. Define $F(y)$ as a CDF of matched men, where $F(y_0) = 0$ and $F(Y) = 1$.

Figure A5: Illustration of surplus difference conditions for each matching equilibria



As labeled in Figure 3, when M-type women match at the top of the distribution, call the lower male income threshold for matching with an M woman y_3. When instead H-type women match at the top, call the male income threshold y_4. When the H-type women match at the bottom of the male income distribution, call the upper male income threshold for matching with an H woman y_1. When instead L-type women match at the bottom, call the male income threshold y_2.

The optimization will be over the bottom man to receive an H-type match: call this $\underline{y}$. Define h as the length of the segment of men who match with H-type women, so that the top man who receives an H-type match will be $\underline{y}+h$.[18] Finally, let $s^K(y)$ represent the surplus obtained from a match with a man of income y and a woman of type $K \in \{L,M,H\}$.

We can now write down the single variable optimization problem to maximize the total surplus

[18] $h = F^{-1}(F(\underline{y})+g^H)-\underline{y}$

by choosing $\underline{y}$:

$$\max_{\underline{y}\in\{y_0,y_4\}}\begin{cases}\max_{\underline{y}\in\{y_0,y_2\}}\int_{y_0}^{\underline{y}}s^L(y)f(y)\mathrm{d}y+\int_{\underline{y}}^{\underline{y}+h}s^H(y)f(y)\mathrm{d}y+\int_{\underline{y}+h}^{y_3}s^L(y)f(y)\mathrm{d}y+\int_{y_3}^{Y}s^M(y)f(y)\mathrm{d}y,\\ \max_{\underline{y}\in\{y_2,y_4\}}\int_{y_0}^{y_2}s^L(y)f(y)\mathrm{d}y+\int_{y_2}^{\underline{y}}s^M(y)f(y)\mathrm{d}y+\int_{\underline{y}}^{\underline{y}+h}s^H(y)f(y)\mathrm{d}y+\int_{\underline{y}+h}^{Y}s^M(y)f(y)\mathrm{d}y.\end{cases}$$

Intuitively, this maximization divides the problem into two cases: one where the segment of men matching with H-type women bisects the segment matching with L-type women (or there is a corner solution), and one where the segment of men matching with H-type women bisects the segment matching with M-type women (or there is a corner solution). Because the surplus gain from switching to an H type is quadratic and concave, we need to find a segment of length h on either side of the maximum benefit from an H match. Thus, the first order conditions for this problem reduce to finding a $\underline{y}$ for which the surplus gain is equal to that of $\underline{y}+h$. When one cannot be found, there is a corner solution, which are the match types 1, 3, and 5. The boundaries for the equilibria are in terms of the surplus gain from switching from either L or M to an H-type woman at the ends of each segment. This provides a full characterization of exactly which form the stable match will take.

The boundaries of each match type in terms of δ_π and δ_γ are shown in Figure A4.

B.3 Equilibrium utilities

This section describes the process for calculating the equilibrium utilities, which are needed to back out the payoff to women of investing in human capital. Because the stable match results from a competitive market, we can recover these utilities as the "prices" associated with each individual. That is, we can calculate the surplus share each individual receives, or the utility over and above their counterfactual single utility.

Because at the stable match the sum of any two individuals' utilities must be greater than or equal to the surplus they could create from marrying one another, we can imagine the matching process as each spouse choosing the partner that maximizes his or her own share of the surplus conditional on keeping his or her spouse happy. That is, for women:

$$v(z,\,p)=\max_{y}\{s(y,\,z,\,p)-u(y)\}.$$

The first order condition of this problem dictates that the slope of the husband's value function must equal the slope of his contribution to the surplus:

$$\begin{aligned} u'(y) &= \frac{\partial s(y,\, z,\, p)}{\partial y} \\ &= \frac{1}{2} p(y+z-1). \end{aligned}$$

Because men's partner type does not change locally with their income except at the "boundaries" of a given female type, we can ignore the woman's type and integrate this function to pin the utility down to an additive constant. Then, we know what men's surplus share will be when matched with each of the three types of women:

$$u^K(y) = \frac{1}{4} p^K y(y+2z^K-2)+\mu^K$$

where $K \in L,\, M,\, H$, and p^K and z^K refer to the fertility and income of a K type woman.

Women's surplus shares will be a constant for each type, v^K. We can solve for each of the constants and the woman's surplus shares using two sets of restrictions. First, that for each couple the two surplus shares must add up to the surplus produced by the match, and second, that for each male type at a "boundary" between two female types, the utility achieved through each match must be the same. This pins down all values except for the division of surplus between the poorest man and his wife.

Assuming initially that there are more men than women in the market provides this restriction, and allows us to assume the poorest man receives no surplus (since otherwise the unmatched men would compete to take his place), and thus $u(y_0)=0$ (with his total utility simply equaling y_0).

I will now go through an example of this process for equilibrium 3, where high-income women are matched with the middle income men, from y_2 to y_3.

The two "boundary" men, y_2 and y_3, must be indifferent between their possible partners, as otherwise the match will not be stable. Thus we know $u^L(y_2)=u^H(y_2)$ and $u^M(y_3)=u^H(y_3)$. This allows us to pin down the constants μ^M and μ^H relative to μ^L (as a function of y_2 and y_3, but recall these are simple functions of the densities of female types, g^K). To pin down μ^L, we use the assumption that there are more men than women, and thus the lowest-income man earns

0 surplus, and thus $u^L(y_0)=0$.

From here, we can solve for the female surplus shares in each pairing, which will each be a constant simply using the total surplus restriction:

$$v^K = s^K(y) - u^K(y).$$

We then have a full characterization of women's and men's surplus shares from marriage, and can further characterize their full utility based on their single utility plus the surplus share, i.e., for men $y+u^K(y)$ and for women z^K+v^K.

Note that a woman's value function responds to fecundity loss through two channels. First, even if the woman's consumption level stayed constant, her utility would be reduced through the lower probability of conceiving, since children directly impact her utility. However, her consumption will also be reduced via the marriage market equilibrium, given that lower fecundity also lowers her husband's utility, and thus he requires a greater share of the available consumption in order to agree to the match.

B.4 Surplus function with multiple children

The following setup expands the model to allow up to four children, with proportionately constrained investments in children for each fertility realization up to four. Let c represent the number of realized children, and investments in children be constrained away from the optimal level by the limited number of children to invest in. Here, I make the strong assumption that there is no reallocation of resources to existing children, but weaker assumptions yield very similar results.

The constrained optimal $q^{'}$ and $Q^{'}$ are:

$$Q_c^{'} = \left(\frac{c}{4}\right)\frac{y+z+1}{2}$$
$$q_c^{'} = y+z-\left(\frac{c}{4}\right)\frac{y+z+1}{2}.$$

Let p_c be the probability of achieving each family size c, where $\sum_{c=0}^{4} p_c = 1$. Then the total household surplus will be the weighted average of all possible family size realizations:

$$s(y,\,z,\,p)=\sum_{c=0}^{4}\left[p_c\left(y+z-\frac{c}{4}\frac{y+z+1}{2}\right)\left(\frac{c}{4}\frac{y+z-1}{2}\right)\right]-y-z \tag{2}$$

This surplus has the same properties as the surplus given in equation (1), including the quadratic form of the difference between matching with a high fertility, low income woman and a low fertility, high income woman. If one had data on desired fertility, an even more flexible model could replace four with the specific number of desired children for a given couple.

C Appendix: Simulation

C.1 Simulation Details

Additional details of the simulation is as follows. For men, total income is drawn from the distribution of married men between 41 and 50 in a given Census year, weighted according to person weights. For women, the parameters are the same, but drawn conditional on education, proportional to the weighted education distribution in each year. An approximate NPV of lifetime income is created using a formula of 20 years of discounted income for women and 25 years for men, with a discount rate of 0.08. Given ages are similar, Income is an approximate sufficient statistic for lifetime income, but this scaling is done because the matching algorithm is not scale invariant, so incomes should be the approximate correct order of magnitude. Other methods of calculating the NPV, such as adjusting for total working years from time of marriage, controlling for age, and using different income paths over time yield highly similar results.

Fertility is assigned according to the empirical distribution of the number of children conditional on educational category and year, but calculated for 38-42 year old women, given that only number of children at home is available in each year, and this can be distorted for older women whose children may have left home. Results for children ever born, available until 1990, are substantively similar.

To determine the matching, a matrix is created of the surplus for matching each man with each woman. The surplus maximizing match is computed using the Hungarian algorithm. Each simulation uses 800 draws of men and women, and the simulation is repeated 10 times.

For the simulation of endogenous education, the number of college educated women is drawn to match the total number of college and highly educated, and they can choose to become highly educated by paying a heterogenous utility cost ranging uniformly from -10^{11} to 2×10^{11}. A matrix is created with the investment and no investment surplus, the maximum chosen, and then the Hungarian algorithm run to decide on matching, which is equivalent to letting couples match first, then choose the optimal investment decision.

C.2 Fertility

The simulation is performed with the number of children currently in the home, for 38-42 year old women. A different age range is used than the age range for the incomes because of the need to minimize bias from children leaving home. Children born, which would not have this bias, is only available up to 1990, as shown in Figure A6. The numbers are very similar to children at home in terms of the very similar numbers of women who have zero children until the highly educated group. Although there is some increase in the number of lower education groups having more than 4 children, it is unclear if these children survived past early childhood. The results in 1990 show the beginnings of the fertility transition shown in the 2010 data, but that there has not yet been full convergence, as is expected.

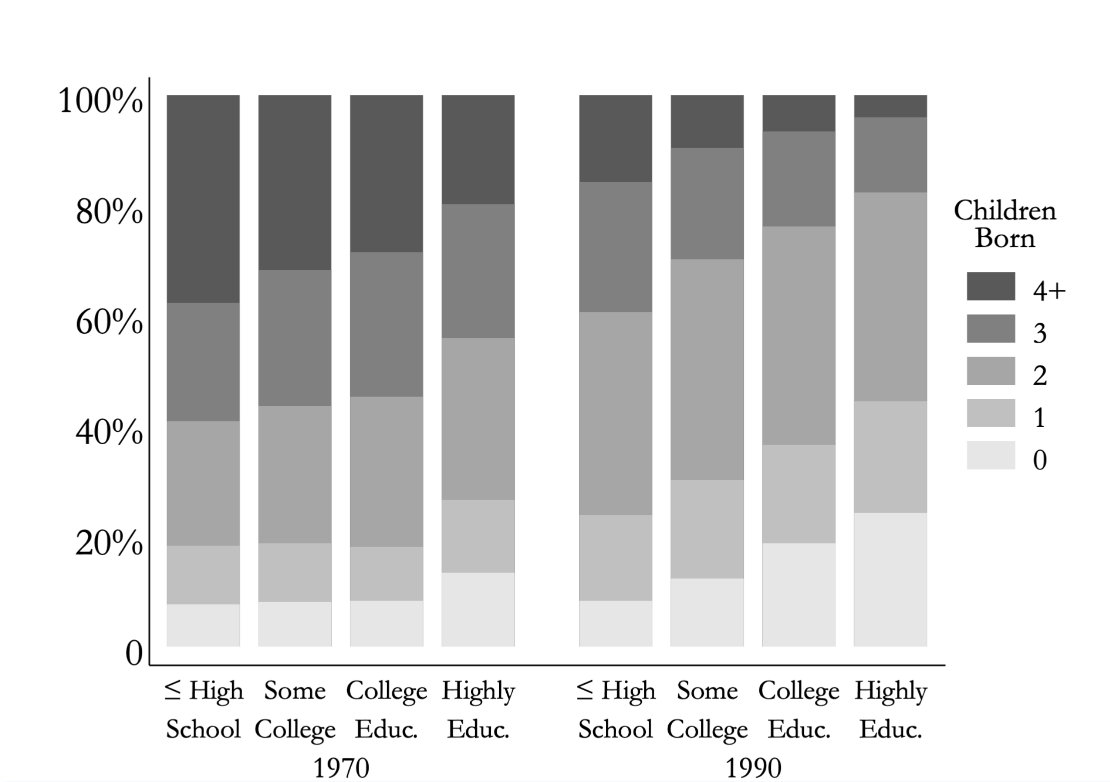
The fertility changes shown in Figure 5 are likely partly attributable in falls in desired family size, shown in A7, which could have driven down college educated women's fertility, making it more comparable to highly educated women's.

C.3 Additional Simulations

Endogenous Education A simulation of matching incorporating endogenous education is shown in Figure A8. It still roughly matches the crossing in the data, but the endogenous education exacerbates the differences between groups, since when one group is more preferred to another, it also tends to be the best women who have selected into being in that group. Introducing additional noise into the decision making and matching process would probably improve the fit.

Marriage and Divorce Though the model was not designed to simulate marriage and divorce rates, it can provide a rough approximation of these metrics by adding a shock that causes couples to either not form or to break up once formed. Who breaks up will be highly dependent on the surplus of the unions they are in, since unions with higher surpluses will be more resilient to higher shocks. Figure A9 shows a simulation of marriage (with divorce having a similar prediction, only

Figure A6: Empirical Distribution of Children Ever Born by Education Level and Year



Notes: Children ever born for women 38-42 years old. Source: 1 percent Census data from 1970 and 1990.

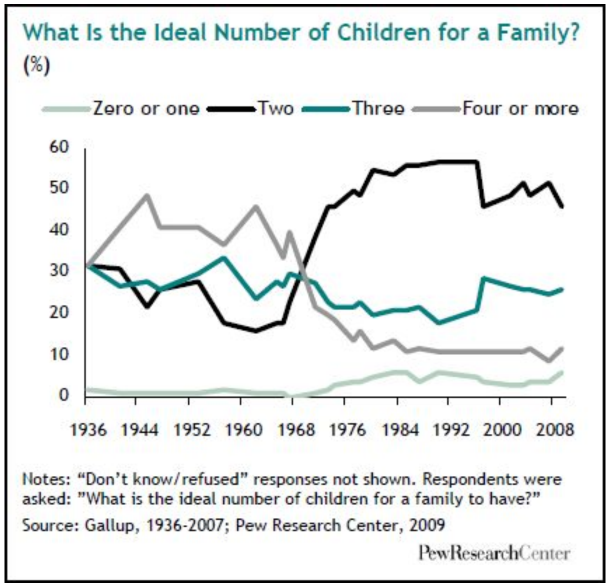
reversed), based on the surplus of hypothetical unions, shown in panel (a), and then shocks drawn from an extreme value distribution, shown in panel (b). Marriage rates are then simulated in panel (c). As in the data shown in Figure A3, highly educated women's marriage rates start below those of college educated women's, and then converge with those of college educated women over time.[19] Thus, this simple model also matches the fact that the "reversal of fortune" for educated women on the marriage market was in fact driven by highly educated women.

C.4 Alternative Explanations

An alternate possible explanation for the change in spousal income by education group over time is that the selection of women into post-bachelor's education has changed in a way that could align with the observed matching patterns. If women previously pursued graduate education

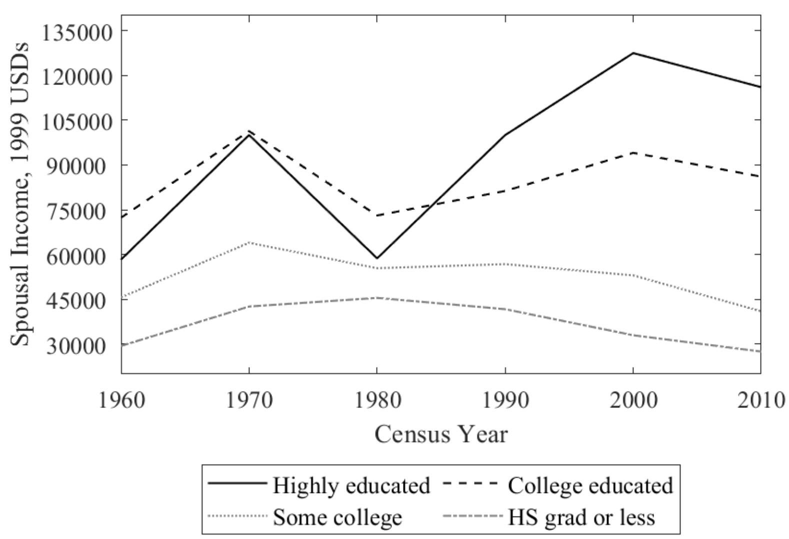
[19] It fails to match the high-until-recently marriage rates of lower educational groups, demonstrating that perhaps cultural factors are needed to explain these groups' marriage behavior.

Figure A7: Desired family size transition



Notes: Figure depicts the rapid transition from four children as the modal desired family size to two children, as evidenced by Gallup polls of men and women. As published in: Pew Center, The New Demography of American Motherhood, August 2010

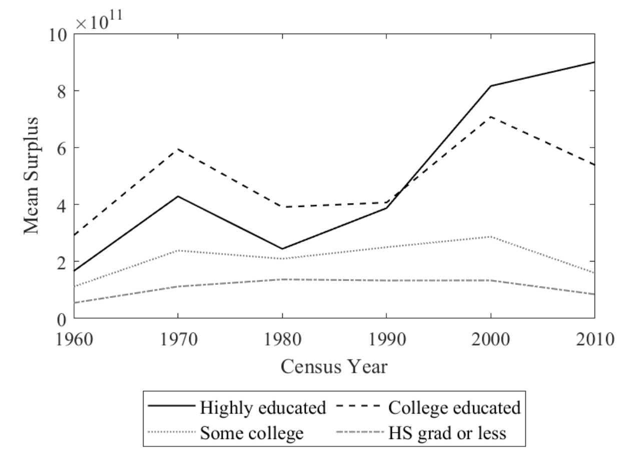
Figure A8: Predicted Spousal Income with Endogenous Education



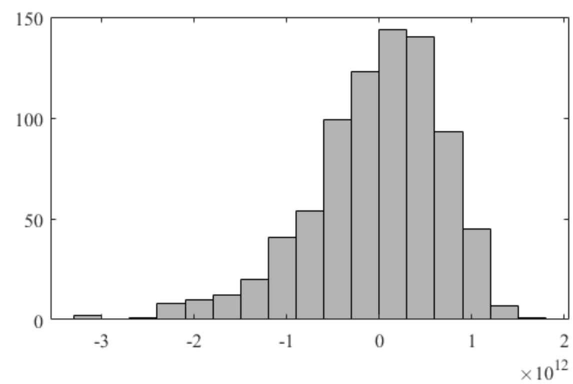
Notes: Model simulation of income matching with endogenous education decision, with uniform education cost, taking NPV of approximate lifetime income conditional on education and fertility as inputs. Matching and education decisions are determined to maximize surplus, as the private education decision will match the efficient equilibrium.

Figure A9: Predicted Marriage Rates with Shocks

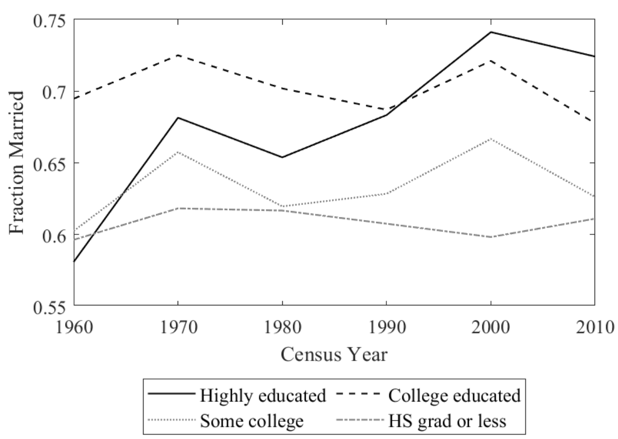
(1) Predicted Surplus



(2) Shocks



(3) Predicted Marriage Rates



Notes: Panel (a) show the total average surplus from the simulation matches shown in Figure 6. Panel (b) shows a distribution of extreme value shocks applied to surpluses to determine whether couples match: couples with negative surplus are assumed to not match. Panel (c) shows the resulting marriage rates by education category.

because they had difficulty marrying, rather than because of higher capability, and this force has lessened over time, one would expect graduate educated women to have been historically less positively selected on skill. I directly test for this using the National Longitudinal Surveys in Appendix Table A2, and show that the "aptitude gap" between graduate educated women and college educated women has remained stable over time.

Table A2 examines whether there has been an increasing skill premium among women who attain post-bachelor's education, using data from aptitude scores and educational attainment of three National Longitudinal Surveys NLS cohorts—the 1968 Young Women panel, the 1979 Youth panel, and the 1997 Youth panel. (Unfortunately, the earliest cohort with the necessary test score information only captures the tail end of the non-assortative matching group, but nonetheless, the trend should be informative about whether there are large shifts occurring in underlying selection factors.) If women were previously selecting into post-bachelor's education due to negative selection in other areas, they may be expected to be less positively selected on intelligence and academic potential. The data shows, to the contrary, that highly educated women indeed had substantially higher aptitude scores on average than college educated women even in the earliest cohort, and that the two numbers are not systematically diverging (which would indicate greater skill-driven selection).

Table A2: Relative college and post-bachelor's average test score percentiles of three NLS cohorts

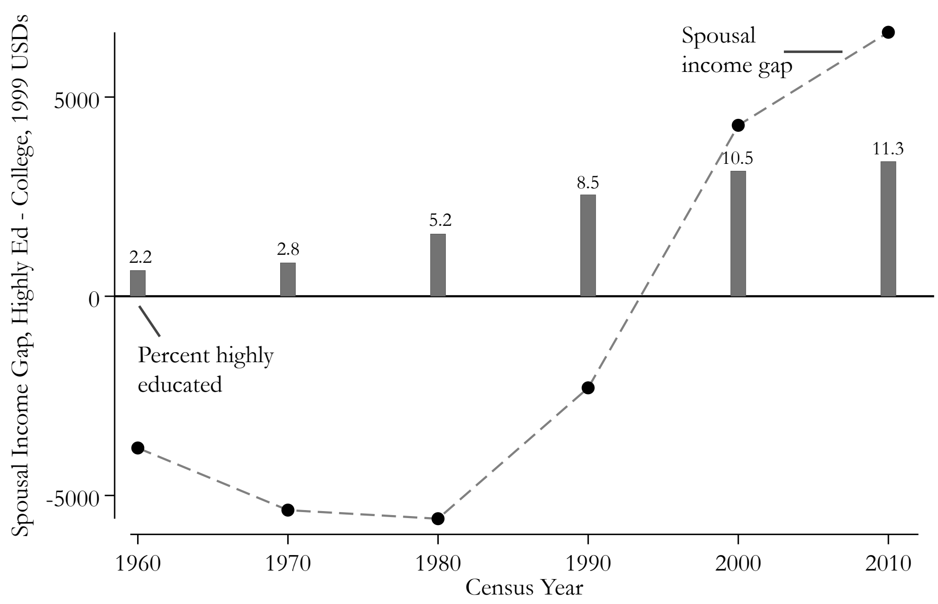
	NLS Young Women 1944-54 birth cohort	NLS Youth '79 1957-64 birth cohort	NLS Youth '97 1980-84 birth cohort
College graduate	66.5	70.3	63.6
Highly educated	72.0	74.9	69.3

Notes: Numbers represent percentiles for test scores by education group, compared to other women of all education levels with test score information available, in three different National Longitudinal Study cohorts. Young Women test score data is from the SAT converted into an IQ measurement. 1979 and 1997 data is from the Armed Forces Qualification Test. Gap in scores between college and graduate-educated women is large and relatively stable. The difference in percentile at both educational levels between the different years may be attributable to score data being available for a different selection of individuals in different survey rounds (e.g., for the Young Women sample, it was only available for individuals who reached the later years of high school).

Additionally, as shown in Figure A10, the spousal income gap between college and graduate-educated women does not respond to the percentage of women earning graduate degrees, as would be expected if the gap were primarily caused by negative selection on other traits. Between the 1970 and 1980 Census, the number of women who achieved post-bachelor's education approximately doubled, while the "penalty" in spousal income compared to college education remained unchanged. From 1990 to 2000 there is a much more modest change in the "pool" of women with graduate degrees, whereas the spousal income gap showed a rapid reversal. This additionally rules out selection in preferences, such as tastes for children. If graduate degrees became less costly, one would expect women with less extreme tastes to join the pool seeking degrees, which would then moderate the spousal income penalty associated with them. However, the spousal income penalty does not appear to respond to the number of women seeking graduate degrees.

Another possible explanation is that highly educated women prefer lower earning partners. Although non-monotonic matching can reflect women's preference for a higher share of the surplus from a lower quality partner, such patterns would be unlikely to arise solely from high-skill women preferring "low-powered" men. If high-skill women would actually rank lower-earning men above higher-earning men (e.g., due to them being able to spend more time at home), the negative-assortative matching at the top would strengthen, rather than weaken, as female earning power grew. Instead, high-skill women may choose a better relative position with a lower quality partner as a compromise because she cannot command a high "bargaining position" (surplus

Figure A10: Rates of women's graduate education versus the spousal income gap



Notes: "Highly educated" constitutes all formal education beyond a college degree. "Spousal income gap" is defined as the average spousal income for highly educated women minus the average spousal income for college educated women. Source: 1 percent Census data from 1970, 1980, and 1990. Sample consists of white women, ages 41-50 years old.

share) when marrying a high-income man. As the reproductive penalty to career investments dissipate, women can realize more equal partnerships with more assortatively matched mates.[20]

D Extension to continuous skill

Rather than having three discrete human capital groups, one could imagine women are endowed with continuous skill, and choose whether to invest in increasing their income relative to their skill. This section briefly outlines the adaptations to the model to accommodate this framework, and demonstrates that results are qualitatively similar to the discrete model.

Setup Men and women are each endowed with skill. In the man's case, human capital investment is assumed to be costless, and he thus arrives on the marriage market with a single characteristic, income, y^h, distributed uniformly on $[1,Y]$.[21]

Women, starting with skill s distributed uniformly on $[0,S]$, can choose to improve their level of income, but doing so takes time, and this time is costly in terms of reproductive capital depreciation. As a result, if they choose to make investments, they will have a lower probability of becoming pregnant when they get married. Women are therefore characterized by a pair of characteristics, (y^w,π). This pair is equal to (s,P) if the woman marries without investing and $(\lambda s,p)$ if the woman marries after investing, where $\lambda > 1$ and $P > p$. Note that the "fertility penalty" of investment is the same for all women, whereas the wage difference from investment increases with skill. Thus, higher skilled women may have more to gain from investing.

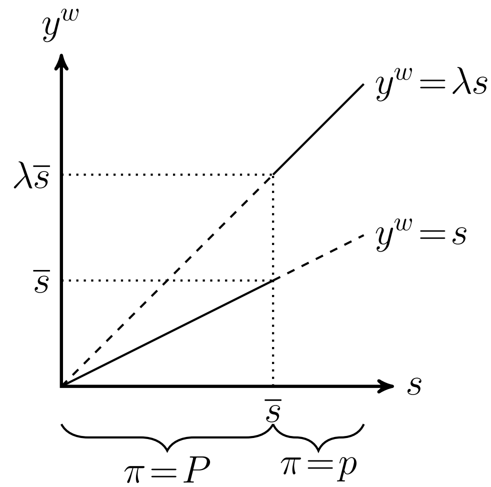
First, I assume an exogenous skill threshold, $\overline{s}$, above which women invest. After determining the equilibrium in the marriage market conditional on $\overline{s}$, I use this equilibrium to solve backwards for which women would optimally invest in the first stage. Thus, assume women with $s > \overline{s}$ invest, earn income of λs, and have fertility p, whereas women with $s < \overline{s}$ earn income s and have fertility P, as shown in Figure A11.

After couples match, each has a child with probability π, and allocates their income. This

[20] Bertrand et al. [2020] offers gender norms against career women as a possible explanation, suggesting these norms may have dissipated in recent years. My model demonstrates, though, that such a norm shift could at least partly be driven by economic fundamentals.

[21] Starting at 1 simplifies the model by ensuring all individuals want to marry, because marriage is only "profitable" if total income is greater than 1.

Figure A11: Women's Income versus Potential Income: Exogenous $\bar{s}$



Notes: Women are endowed with skill, s, shown on the x-axis. Their level of income, y^w, shown on the y axis, is determined by their investment decision. If women invest, they earn income λs, with $\lambda > 1$, but at the cost of reducing their fertility, π from P to $p < P$. In this section, we assume women with $s > \bar{s}$ invest.

process determines the surplus created by a given marriage, and thus individuals' preferences over different matches. Thus, solving the model requires working backwards from consumption decisions if a child occurs, to the surplus function from marriage, to then determining the optimal match.

As before, married couples can spend income on private consumption given by q^h and q^w and a public good, investment in children, denoted by Q:

$$U^h(q^h, Q) = q^h(Q+1)$$
$$U^w(q^w, Q) = q^w(Q+1),$$

with budget constraint $q^h + q^w + Q = y^h + y^w$

Optimal consumption is given by:

$$q^* = \frac{y^h + y^w + 1}{2}$$
$$Q^* = \frac{y^h + y^w - 1}{2}.$$

(Corner solutions are avoided by restricting $y^h + y^w > 1$ based on the distributions of y and s).

The joint expected utility from marriage, T, is a weighted average between the optimal joint utility if a child is born and the fallback position of allocating all income to private consumption:

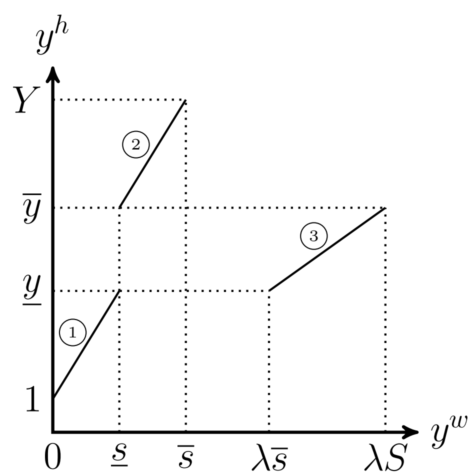
$$T(y^h,y^w,\pi)=\pi\frac{(y^h+y^w+1)^2}{4}+(1-\pi)(y^h+y^w).$$

And the surplus function is similarly:

$$s(y^h,y^w,\pi)=\pi\frac{(y^h+y^w-1)^2}{4}.$$

Equilibrium An equilibrium displaying assortative matching for women with the same fertility, but non-assortative matching for women with different fertility levels, is shown in Figure A12. Recall $\overline{s}$ is the skill threshold for women becoming educated. Poor men, from 1 to $\underline{y}$, marry low-skill, fertile women (matching assortatively). On the other side of the threshold, the richest group of women matches assortatively with the middle group of men, from $\underline{y}$ to $\overline{y}$. But the richest men, from $\overline{y}$ to Y, forego matching with the richest women and instead marry the "best of the rest"—the more high-skilled women among those who have not invested and are thus still fertile.[22]

Figure A12: Non-monotonic Equilibrium Match



Notes: Women's income, y^w is on the x-axis, and men's income, y^h on the y-axis. The diagonal lines represent matching between men and women. In this non-monotonic matching equilibrium, women with income between 0 and $\underline{s}$ match with men with income between 1 and $\underline{y}$. Women with incomes between $\underline{s}$ and $\overline{s}$ match with men with incomes between $\overline{y}$ and Y. Women who have invested, and thus have incomes between $\lambda\overline{s}$ and λS, match with men with incomes between $\underline{y}$ and $\overline{y}$.

[22]The matching functions in this uniform case are linear—in an arbitrary distribution, their form would be determined by the density of individuals, so that the number of women above any point exactly matches the number of men above that point.

The equilibrium value functions can be used to show that this is indeed a stable match when λ, the income gain from investing, is high enough to overcome the fertility cost, $\frac{P}{p}-1$, for some men, but not high enough that all men prefer women who have invested. In particular, when $\frac{S-\bar{s}}{S+\bar{s}}(\frac{P}{p}-1)\frac{Y-1}{S}<\lambda<(\frac{P}{p}-1)\frac{Y-1}{S}$, the three-segment match is the unique stable match. For *any* value of S, $\bar{s}$, P, and p, such a λ exists, as $\frac{S-\bar{s}}{S+\bar{s}}<1$. Thus, this model predicts non-monotonic matching.

The matching equilibrium implies that as λ grows relative to $\frac{P}{p}$, the world transitions from one where educated women are penalized for their investment, because the additional income they earn is insufficient to compensate wealthy male partners for their loss in fertility, to one where they are able to compensate, and thus match with, partners similarly high in the income distribution.

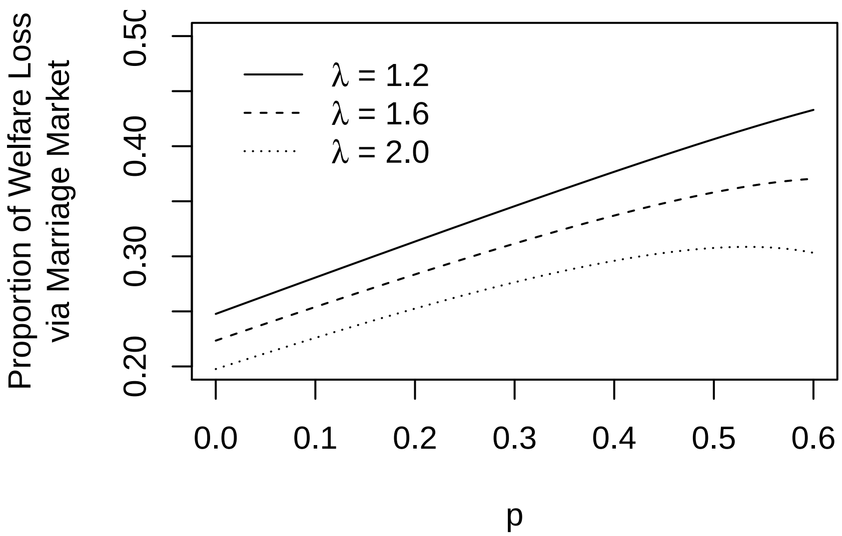
The lower bound on women's skill for them to be willing to invest, $\bar{s}$, can be found by using the payoff functions resulting from the matching equilibrium, and finding the point at which the investment payoff dominates the non-investment payoff, with a small fixed cost to investment (note, as here the cost to invest is in this setup a monetary, rather than utility cost, it is also possible that very high-skilled women choose not to invest—i.e., non-monotonicity in investment decisions). To simplify this section, let $Y=2$, $S=1$, and $P=1$.

Using the equilibrium payoff functions, we seek the skill level at which $v_3(\bar{s})=v_2(\bar{s})$, or $\bar{s}^*(\lambda,c,p)$. Although its functional form is complex, $\bar{s}^*$ varies with the parameters in expected ways: it is increasing in c, decreasing in λ, and decreasing in p. In other words, the higher the fixed cost of investment, the higher the skill threshold for pursuing it; the higher the return to investment, the lower the skill threshold; and, the higher the chance of conceiving following investment, the lower the threshold. A higher threshold means fewer women making career investments. A lower threshold means more women making career investments, and this can be spurred by a lower fixed cost of investment, greater returns, or a higher chance of conceiving (e.g., through IVF technology).

Note that the equilibrium payoff function internalizes not just the individual change in utility from a different fertility level, but also any change in the share of surplus received. This reflects the impact of traits on the overall surplus: someone with traits that yield a large surplus will in exchange receive a favorable match with a high surplus share. Someone with less desirable traits will face a less desirable match and a lower surplus share. Thus, when equalizing the payoff between investing and not investing to find the optimal threshold, both the personal cost of lower fertility and the cost to the marital surplus are considered

Welfare Crucially, the model provides a mechanism through which the biological clock impacts women's welfare through a channel other than her own desire for children. That is, even if a woman did not care at all about having a family, she would still be negatively impacted by her fertility loss through her loss of status on the marriage market.

Figure A13: Proportion of Welfare Loss from Time-Limited Fertility due to Marriage Market



Notes: Figure depicts the portion of the welfare loss (y-axis) from lower fertility that comes through the marriage market compared to the total welfare loss, for varying values of λ, across a range of values for p (x-axis). This is shown for the most-skilled woman, with skill-level S, across the range of ps where non-monotonic matching results, for parameter values $Y = 2$, $S = 1$, and $P = 1$, with an exogenous t, investment threshold, of 0.7. The graph is produced by calculating the change in woman's indirect utility between a scenario with zero fertility cost of investment, to one where post-investment fertility equals p, and comparing that to the same effect if her partner and share of the marital surplus were held constant.

In fact, a back of the envelope calculation using the model suggests that approximately one-third of the utility cost from the post-investment fertility loss comes through the marriage market, rather than directly through women's utility over children. Figure A13 compares the utility loss of lower fertility from the marriage market alone to the loss including women's personal valuation of fertility. The portion of the welfare loss stemming from being matched with a lower quality spouse and needing to cede more of the marital surplus to that spouse ranges from 20-40% of the total utility cost.[23] This simple calculation highlights that the loss of reproductive capital

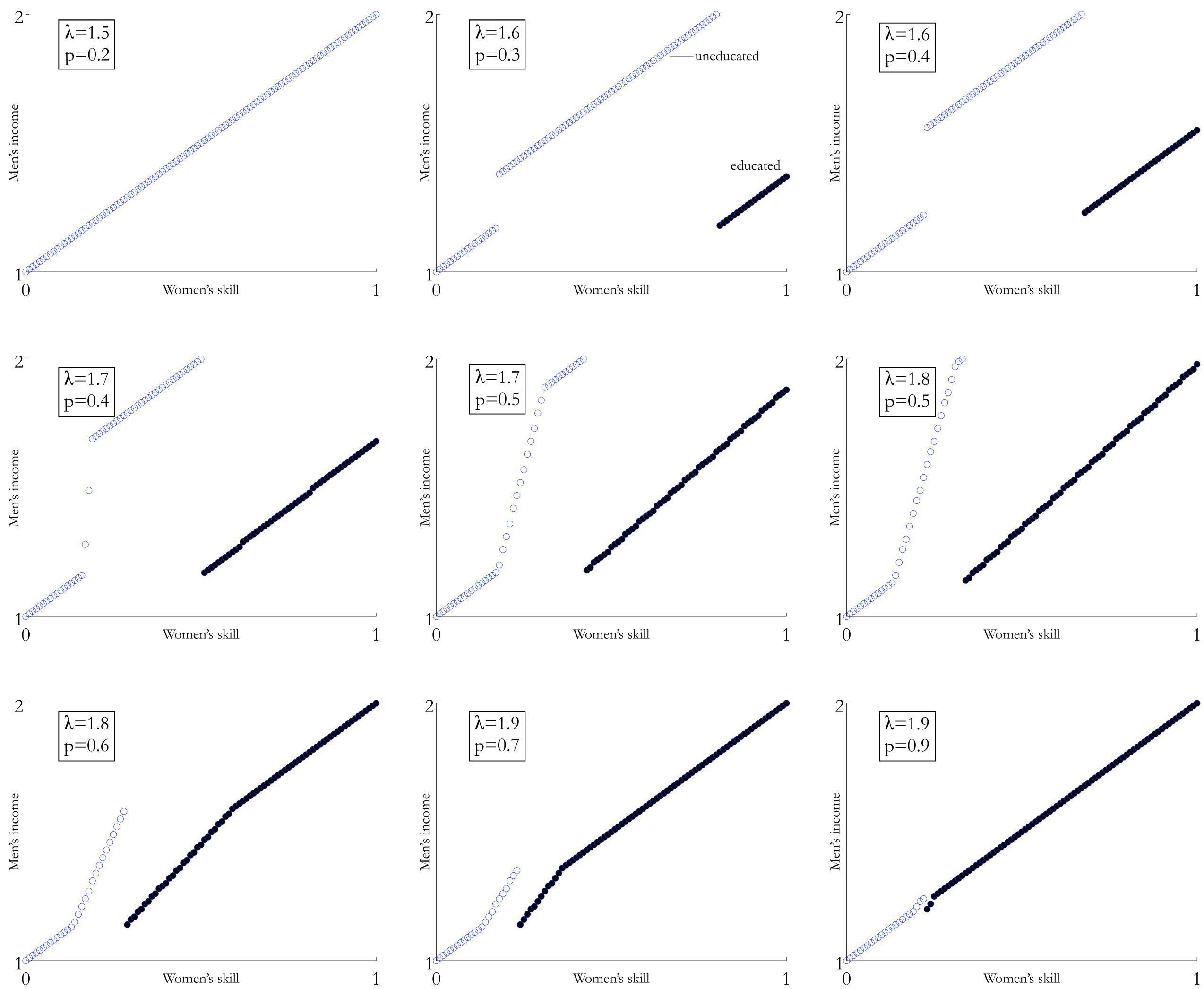
[23] The calculation is $1 - \frac{v(S|p=P) - v(S|MM)}{v(S|p=P) - v(S)}$ where $v(S|p=P)$ is the woman with skill S's indirect utility if she invests but fertility is unaffected, $v(S)$ is her actual indirect utility, and $v(S|MM)$ is her indirect utility if there

is an economic loss, just as worker disability is. Because the marriage market creates value for women, the loss of a valuable asset on that market creates real economic impacts.

Simulation of continuous model Figure A14 simulates the model in the presence of growing returns to women's education and falling fertility costs. The first row of images in Figure A14 show that at first, no woman is willing to risk the marriage market costs of investing, so human capital accumulation by women is limited, and matching is assortative. As λ, the gain from investing, slowly increases while the fertility cost falls (via increasing the success of post-investment conception, p), the education and marriage market transforms. The first women to invest, shown by dark blue dots, are penalized through worse marriage matches, creating the non-monotonic equilibrium exhibited in the early Census data. Over time, as labor market returns to investment rise and the fertility cost falls, the marriage matches of these women gradually improve, as seen in the second row of images. This, in turn, creates a feedback loop, with more women being willing to invest (which also matches the dramatic rise in US women pursuing higher education). Finally, the market becomes essentially assortative, with some "randomization" by the highest earning men: some marry the very richest women, while others still choose the best among the women who have not invested.

is indeed a fertility penalty, but she were to still match with man Y and receive the same *share* of the surplus as if there were no fertility loss.

Figure A14: Full Two-Stage Optimization Simulation



Notes: Figure depicts the results of a simulation of the investment and matching equilibrium as the value of the return on investment, λ, and post-investment chance of fertility, p, increases. Women's skill is shown on the x-axis and men's income on the y-axis, with dots depicting marriage matches. At first, the returns are low enough—and the potential marriage market cost high enough—that no women invest (and thus matching is assortative). As λ and p rise, some women invest (shown by dark blue dots, but these top-skilled women are penalized on the marriage market, and matching is non-monotonic. As λ and p continue to grow, matching becomes more assortative. Simulation shown for Y=2, S=1, P=1, and c=0.2.